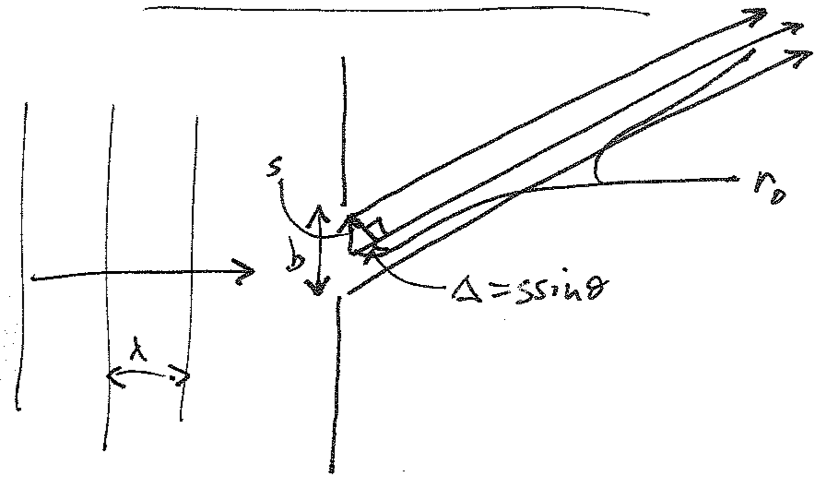
Single Slit Diffraction



Fraunhofer Diffraction (Far Field): Observe interference at infinity as a function of θ. Far field condition: $L \gg \frac{b^2}{\lambda}$

Consider each point in the aperture a source of spherical waves.

Field at distant point P due to a single point source on the aperture:

$$dE_p = \underbrace{\left(\frac{E_L\, ds}{r}\right)}_{\text{Amplitude factor for spherical waves}} \underbrace{e^{i(kr-\omega t)}}_{\text{phase factor}}$$

$\left(I \propto \frac{1}{r^2}, \text{ so } E \propto \frac{1}{r} \right)$

Let r_0 be the distance travelled by the center wavelet.

Then $\Delta = s\sin\theta$

And $dE_p = \frac{E_L\, ds}{(r_0+\Delta)} e^{i(k(r_0+\Delta)-\omega t)} \approx \frac{E_L\, ds}{r_0} e^{i(kr_0-\omega t)} e^{ik\Delta}$

can ignore Δ compared to r_0

Can't ignore phase due to Δs

Integrate of the aperture (s) to get the total field at point P:

$$E_p = \int dE_p = \frac{E_L}{r_0} e^{i(kr_0 - \omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{ik\Delta} ds \qquad \Delta = s\sin\theta$$

$$= \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \left. \frac{e^{iks\sin\theta}}{ik\sin\theta} \right|_{-b/2}^{b/2}$$

$$= \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \left[\frac{e^{ikb\sin\theta/2} - e^{-ikb\sin\theta/2}}{ik\sin\theta} \right]$$

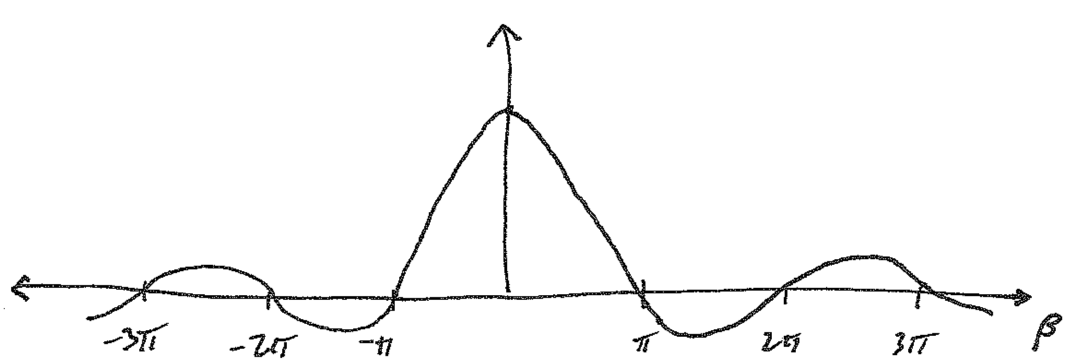
Define $\beta = \frac{1}{2} kb\sin\theta$

$$E_p = \frac{E_L b}{r_0} e^{i(kr_0-\omega t)} \left(\frac{\sin\beta}{\beta} \right)$$

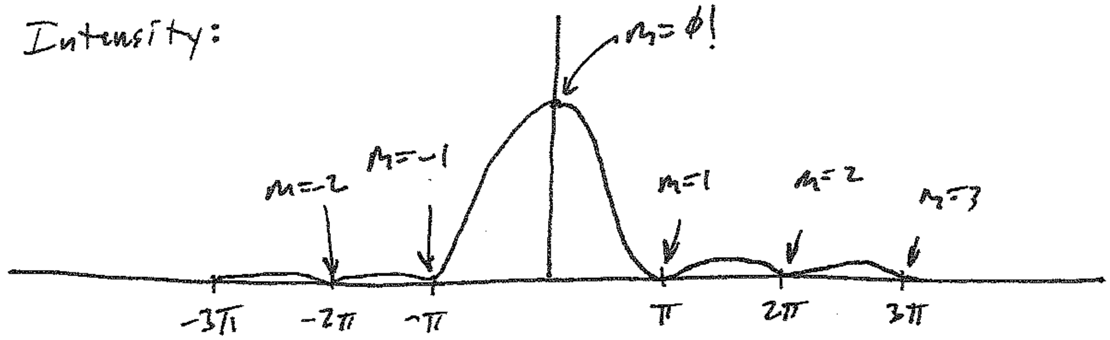
$$\boxed{I \propto E_p E_p^* = \left(\frac{E_L b}{r_0}\right)^2 \frac{\sin^2\beta}{\beta^2}}, \quad \beta = \frac{1}{2}kb\sin\theta.$$

$$I = I_0 \frac{\sin^2\beta}{\beta^2}$$

Electric Field Amplitude:



Intensity:



Zeros occur when $\beta = \pm m\pi$, $m = \pm 1, \pm 2, \pm 3$

But not $m = 0$!

$\frac{1}{2} k b \sin\theta = m\pi$

$\frac{b}{\lambda} \sin\theta = m$

$\boxed{m\lambda = b\sin\theta}$ zeros, $m = \pm 1, \pm 2, \pm 3, \ldots$

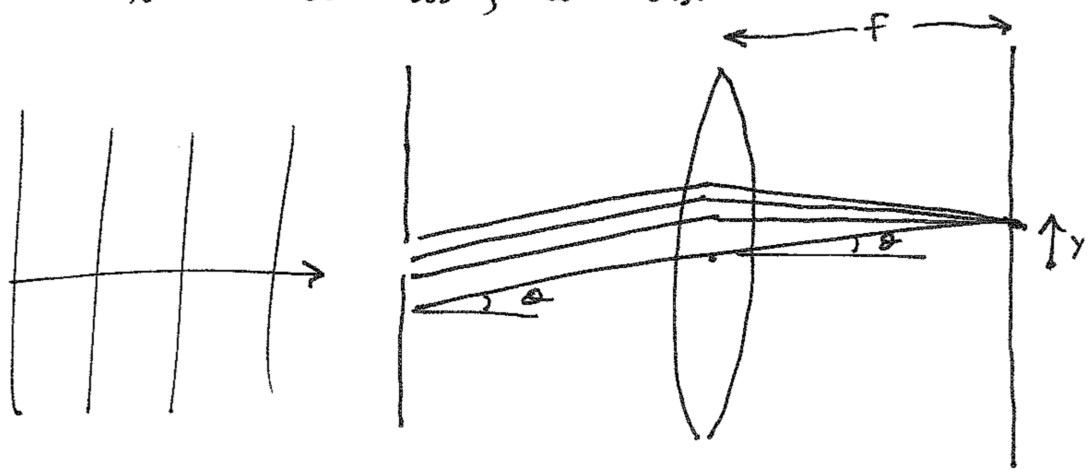
If θ = small, $\sin\theta \approx \theta$, width of central peak:

$m = 1$: $\frac{b\theta_1}{\lambda} = 1$

$m = -1$: $\frac{b\theta_2}{\lambda} = -1$

$\boxed{\Delta\theta = \frac{2\lambda}{b}}$

Also, we can move the diffraction pattern from infinity to a screen using a lens:



Can write θ in terms of position on screen:

$$\frac{y}{F} = \tan\theta \approx \sin\theta$$

$$\sin\theta = \frac{y}{F}$$

$$m\lambda = \frac{by}{F}$$

$\boxed{y_m = \frac{m\lambda F}{b}}$ zeros on a screen with a lens.