Recap from last week

Index of refraction : $n = \frac{c}{v}$

Snell's Law : $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Refraction)

Law of Reflection : $\theta_1 = \theta_2$

Total internal reflection : If $n_2 > n_1$, then $\sin\theta_c = \frac{n_2}{n_1}$. If $\theta > \theta_c$ no light is refracted, 100% reflected

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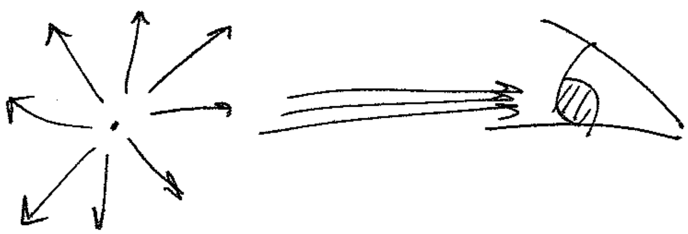
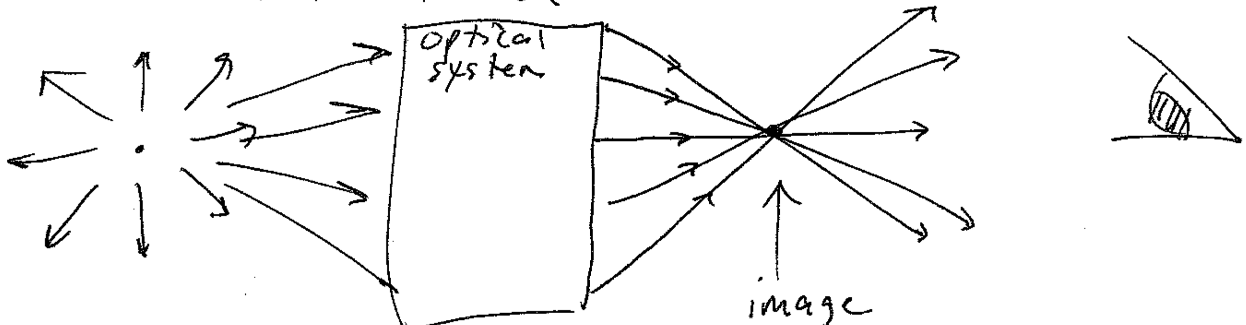
Imaging:

Most objects are rough - they scatter light in all directions



Human eye collects a small fraction of the scattered light.

An optical system can re-direct some of the rays to a 2nd location, simulating what the object would look like if it were there:



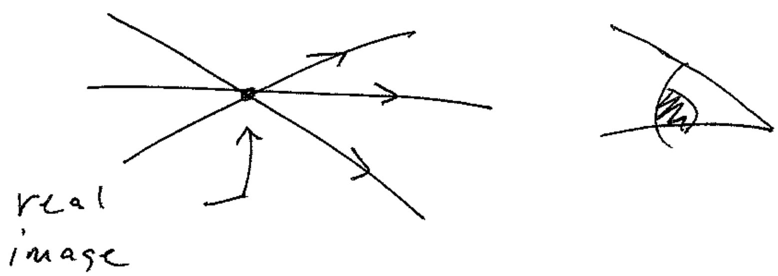
Two ways to see an image:

1) Look down the optical axis so the rays from the image enter your eye : magnifier, microscope, telescope.

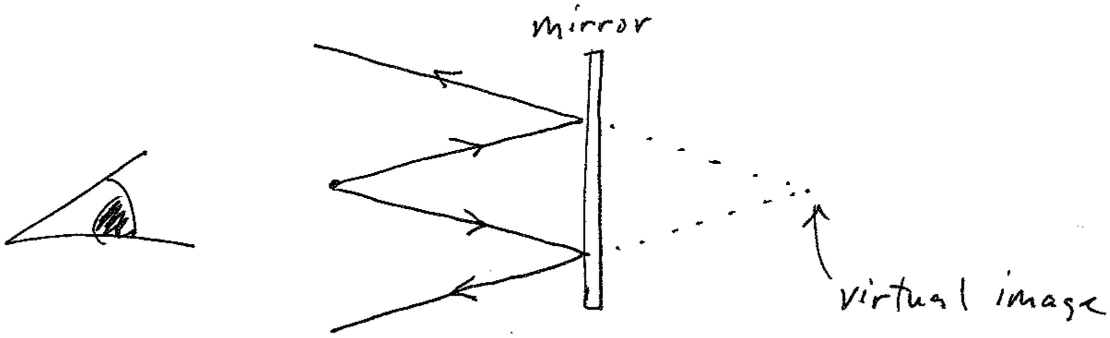
2) Use a screen ~~to~~ at the image location to scatter the rays in all direction. (Then you can see the image from any angle.)

<u>Two types of Images</u>

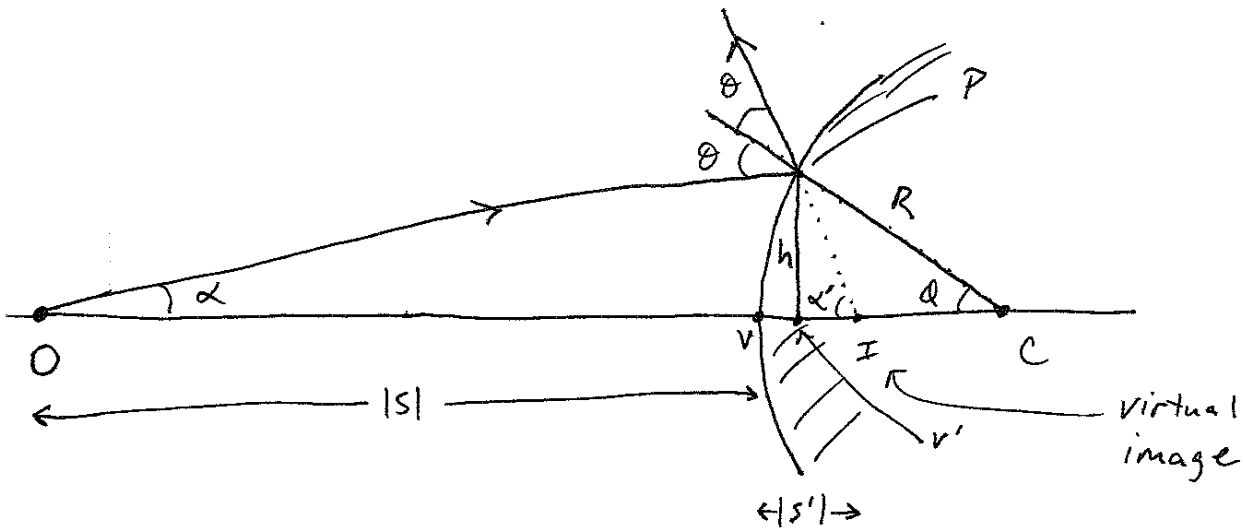
Real: The rays actually pass through the image point:



Virtual: The rays appear to pass through the image point, but they don't:



Spherical Convex Mirror



Triangle OPC: $\alpha + \varphi + (\pi - \theta) = \pi \Rightarrow \alpha + \varphi = \theta$ ①

Triangle OPI: $\alpha + \alpha' + (\pi - 2\theta) = \pi \Rightarrow \alpha + \alpha' = 2\theta$ ②

$2 \times$ ① $-$ ②: $\alpha + 2\varphi - \alpha' = 0$

$$\boxed{\alpha - \alpha' = -2\varphi}$$

Small angle approximation: $\sin\theta \sim \theta \sim \tan\theta$, $\cos\theta \sim 1$

$\tan\alpha \sim \alpha = \frac{h}{|S|}$ (ignore small distance VV')

$\tan\alpha' \sim \alpha' = \frac{h}{|S'|}$

$\sin\varphi \sim \varphi = \frac{h}{|R|}$

$$\therefore \frac{h}{|S|} - \frac{h}{|S'|} = \frac{-2h}{|R|}$$

$$\boxed{\frac{1}{|S|} - \frac{1}{|S'|} = \frac{-2}{|R|}}$$

Spherical Convex Mirror.

Spherical Concave Mirror

$$\frac{1}{|s|} + \frac{1}{|s'|} = \frac{2}{|R|}$$

Ex: $s = 4$ cm
$R = -2$ cm
$\frac{1}{4} + \frac{1}{s'} = \frac{2}{(-2)} = -1$
$s' = -\frac{4}{5}$

Both concave and convex described by

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{+2}{R}}$$ Spherical mirror

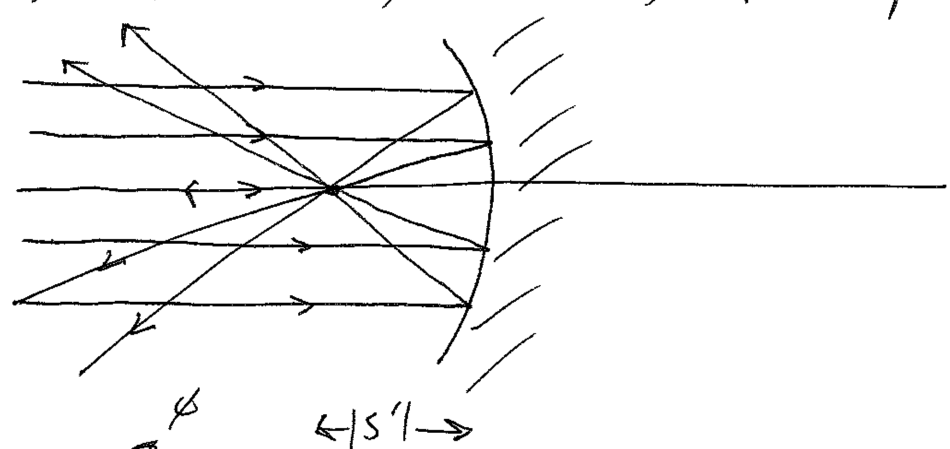
With these sign conventions:

1) object distance s is positive if object is real.
2) image distance s' is positive if image is real.
3) R positive for ~~convex~~ concave mirror, R negative for ~~concave~~ convex mirror.

Need to know if object and image are real or virtual before applying equation

Focal points

If object is at $+\infty$, all incoming rays are parallel:

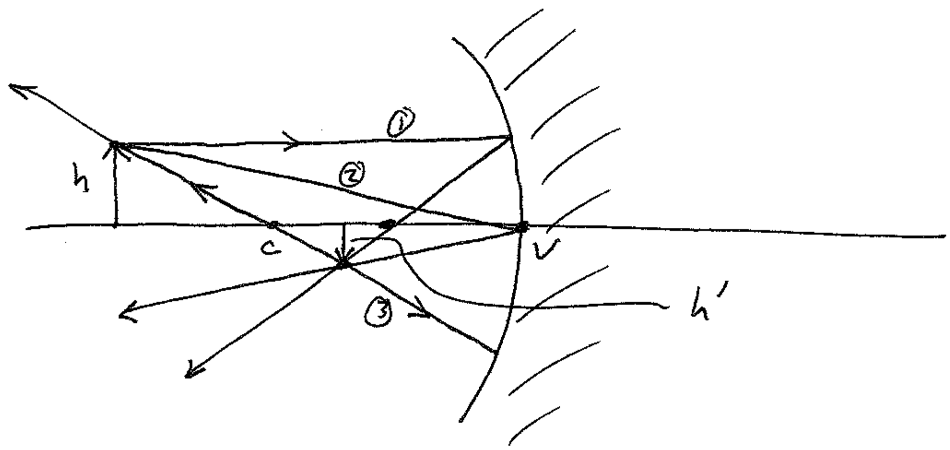


$$\frac{1}{s} + \frac{1}{s'} = \frac{+2}{R}$$ (with $\frac{1}{s} \to 0$)

$$s' = \boxed{\frac{+R}{2} \equiv f}$$ (focal length)

$f > 0$ concave
$f < 0$ convex

Objects Off-axis: non-point objects



Principle Rays:

1) Ray parallel to optical axis passes through focal point after reflection.

2) Ray intersects vertex V reflects at equal angle.

3) Ray passes through center of curvature returns along same path

Magnification: ratio of image height to object height

Similar triangles

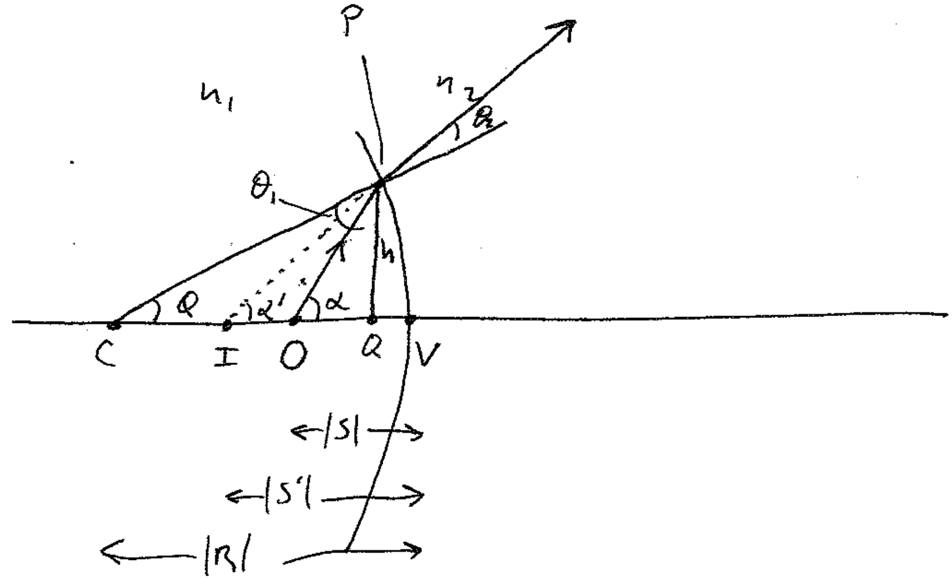
$$\frac{h}{s} = \frac{h'}{s'}$$

$$|m| = \left|\frac{s'}{s}\right| = \left|\frac{h'}{h}\right|$$

Sign Convention m is negative iff image is inverted.

$$\boxed{m = -\frac{s'}{s}}$$

Refraction at Concave Spherical Surface



Triangle CPO: $\varphi + \theta_1 + \pi - \alpha = \pi \Rightarrow \theta_1 = \alpha - \varphi$

CPI: $\varphi + \theta_2 + \pi - \alpha' = \pi \Rightarrow \theta_2 = \alpha' - \varphi$

Snell's Law for small angles: $n_1 \theta_1 \sim n_2 \theta_2$

$$n_1 (\alpha - \varphi) = n_2 (\alpha' - \varphi)$$

Ignore short distance QV:

$$\alpha \sim \frac{h}{|s|}, \quad \alpha' \sim \frac{h}{|s'|}, \quad \varphi \sim \frac{h}{|R|}$$

$$n_1 \left(\frac{h}{|s|} - \frac{h}{|R|} \right) = n_2 \left(\frac{h}{|s'|} - \frac{h}{|R|} \right)$$

$$\boxed{\frac{n_1}{|s|} - \frac{n_2}{|s'|} = \frac{n_1 - n_2}{|R|}}$$

Refraction at Concave Spherical Surface

Re-do for convex surface: $\frac{n_1}{|s|} + \frac{n_2}{|s'|} = \frac{n_2 - n_1}{|R|}$

Both cases: $\boxed{\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}}$ with

1) real objects: $s > 0$
2) real image: $s' > 0$
3) Concave: $R < 0$. convex: $R > 0$