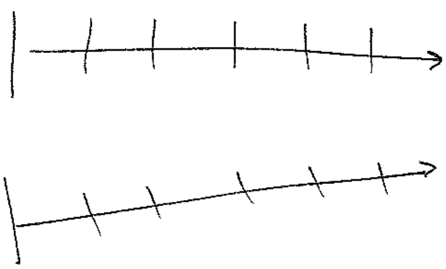
Lecture 8 - Two Beam Interference Phys 375 10/23/0?



$E_1 = A e^{i(kx_1 - \omega t)}$

P

- waves overlap here

$E_2 = A e^{i(kx_2 - \omega t)}$

$E_P = E_1 + E_2$

$I \propto E_P E_P^*$

$= 4A^2 \cos^2 \frac{k\delta}{2}$, $\delta = x_2 - x_1$ = path difference.

More generally, $E_P E_P^* = 4A^2 \cos^2 \frac{\Delta\phi}{2}$,

$\Delta\phi$ = phase difference between beams.

For above case, $\Delta\phi = 2\pi \left(\frac{x_2 - x_1}{\lambda} \right) = k\delta$, $k = \frac{2\pi}{\lambda}$

To observe interference we need

- same polarization
- coherent waves

} use the same light source to meet these conditions

Real light waves:
Not perfect cosine func.

"coherence time" = length of time wave remains predictable

"coherence length" = coherence time × speed of light

He-Ne lasers: $\tau_c \sim 1\,ns - 30\,ns$

$l_c \sim 20\,cm$ to $10\,m$

White light: $l_c \sim \frac{1}{2}$ wavelength $\sim 200\,nm$

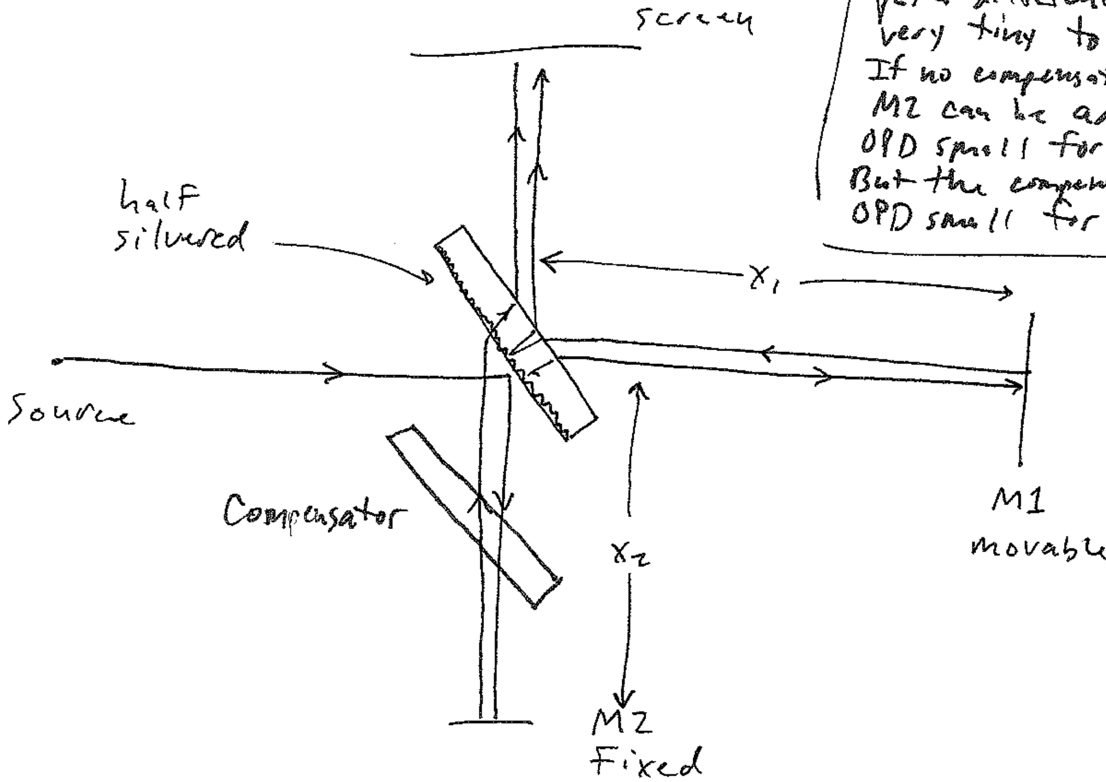
$\tau_c \sim 1 \times 10^{-15}$ sec.

Michelson - 1852 - 1931
First American Nobel Prize - 1907

Michelson Interferometer

Compensator:

For white light, the optical path difference must be very tiny to see interference. If no compensator exists, then M2 can be adjusted to make OPD small for one wavelength. But the compensator makes OPD small for all wavelengths



$$E_p E_p^* = 4A^2 \cos^2 \frac{\Delta\phi}{2}$$

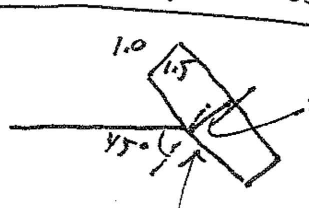
path difference $= 2(x_1 - x_2) = \delta$

phase difference due to path difference $= k\delta = 2k(x_2 - x_1)$

phase difference due to ~~internal~~ external reflection of beam along M2 $= \pi$

total phase difference $= 2k(x_2 - x_1) + \pi = \Delta\phi$

For constructive interference,



@ 45°, both s & p have a π phase shift for an external reflection

@ 28°, both s & p have no phase shift for an internal reflection

$$\frac{\Delta\phi}{2} = m\pi, \quad m = 0, \pm 1, \pm 2, \ldots$$

$$k(x_2 - x_1) + \frac{\pi}{2} = m\pi$$

$$\frac{2\pi}{\lambda}(x_2 - x_1) + \frac{\pi}{2} = m\pi$$

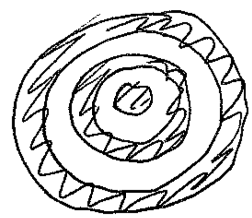
"fringe order"

$$\boxed{\frac{2(x_2 - x_1)}{\lambda} = m + \frac{1}{2}}$$

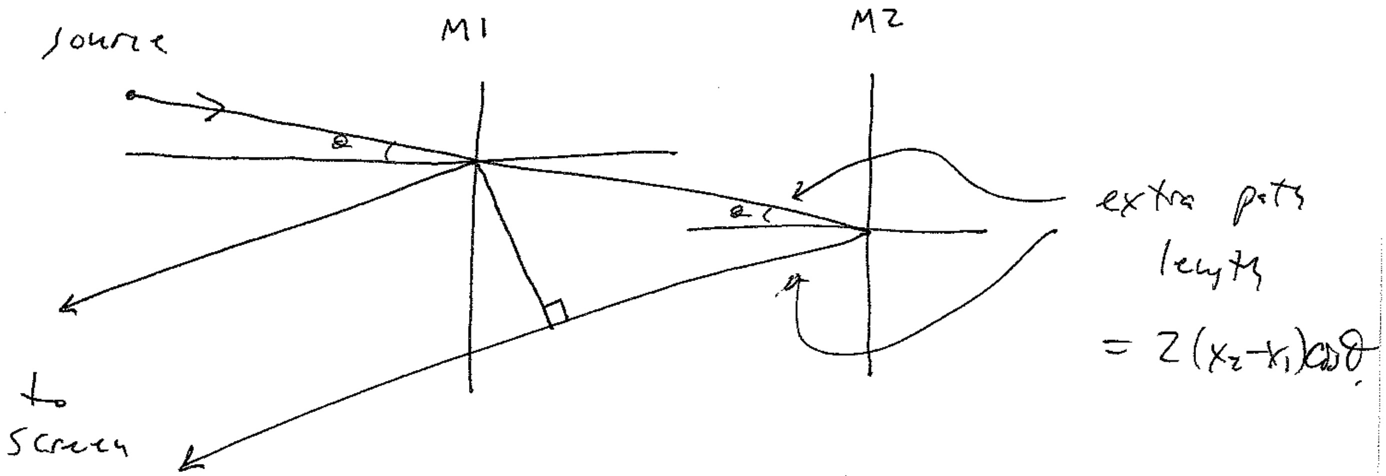
constructive interference

$$\boxed{\frac{2(x_2-x_1)}{\lambda} = m}$$ destructive interference.

In practice you don't see one spot, you see:



Because off-axis points have an extra path difference



If $x_2 \equiv x_1$, then only one fringe is observed for all θ (no off axis phase difference)

Otherwise many rings will be seen.

Usually, we don't care about which order of fringe we are observing. Instead, move M2, and count the number of fringes that pass by:

$$\boxed{\Delta m = \frac{2(x_2-x_1)}{\lambda}}$$

Measure $x_2 - x_1$, count Δm, and calculate the wavelength λ.

Sodium Lamp

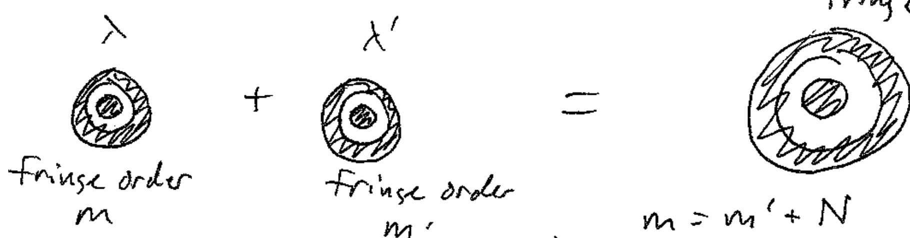
Two closely spaced wavelengths: λ, λ'

Each makes its own fringe pattern.

Suppose both patterns have a maximum:

high fringe visibility

$m = 1000$
$m' = 1500$



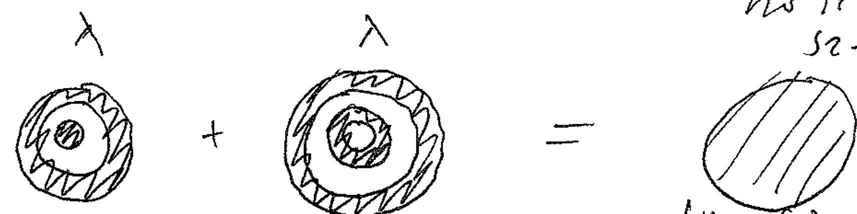
, $m = m' + N$

Then $$\frac{2d_1}{\lambda} = \frac{2d_1}{\lambda_1} + N \qquad d_1 = x_2 - x_1$$

Now move the mirror to increase d. ~~After~~ a while, the two patterns look like

no fringes seen

$m = 1300$
$m' = 1800.5$



low fringe visibility

Then move the mirror more, and the fringes re-appear:

Now $\quad x_2 - x_1 = d_2$

$$\frac{2d_2}{\lambda} = \frac{2d_2}{\lambda'} + \underbrace{N+1}_{\text{m and m' differ by N+1 now}}$$

$$\frac{2(d_2 - d_1)}{\lambda} = \frac{2(d_2 - d_1)}{\lambda} + 1$$

$$\frac{2\Delta d}{\lambda} - \frac{2\Delta d}{\lambda'} = 1$$

$$\lambda - \lambda' = \frac{\lambda\lambda'}{2\Delta d}$$

or, since $\lambda \approx \lambda'$,

$$\lambda - \lambda' = \boxed{\Delta\lambda \approx \frac{\lambda^2}{2\Delta d}}$$

Δd is the change in the mirror distance between the two "high fringe visibility" regions.