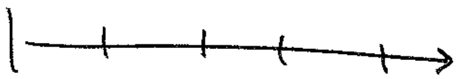
Interference of Light

$$E_1 = A e^{i(ks_1 - \omega t)}$$

$$E_2 = A e^{i(ks_2 - \omega t)}$$



Waves overlap here

- P

To get interference, we need

- same polarization
- coherent waves

} often we get this by using the same light source for both waves.

Assume polarization is the same, so we ignore the vector notation and just use scalar waves.

$$E_p = A e^{i(ks_1 - \omega t)} + A e^{i(ks_2 - \omega t)}$$

$$= A e^{-i\omega t}\left(e^{iks_1} + e^{iks_2}\right)$$

Define $\delta = s_2 - s_1$, $\quad \bar{s} = \frac{1}{2}(s_1 + s_2)$

δ is the spatial difference between wavefronts.

Then $s_1 = \bar{s} - \frac{\delta}{2}$, $\quad s_2 = \bar{s} + \frac{\delta}{2}$

$$E_p = A e^{-i\omega t}\left(e^{ik(\bar{s} - \frac{\delta}{2})} + e^{ik(\bar{s} + \frac{\delta}{2})}\right)$$

$$= A e^{i(k\bar{s} - \omega t)}\left(e^{ik\delta/2} + e^{-ik\delta/2}\right)$$

$$= 2A e^{i(k\bar{s} - \omega t)} \cos\left(\frac{k\delta}{2}\right)$$

$$E_p = \underbrace{2A\cos\left(\frac{k\delta}{2}\right)}_{\text{Amplitude Factor}} \underbrace{e^{i(k\bar{s}-\omega t)}}_{\text{wave factor}}$$

usually (unobservable)

$$I \propto E_p E_p^* = 4A^2 \cos^2\left(\frac{k\delta}{2}\right)$$

↑ gets rid of the wave factor

If we only had one beam, then

$$I = I_p = E_1 E_1^* = A^2$$

So $4A^2\cos^2\left(\frac{k\delta}{2}\right)$ has <u>four</u> times the amplitude of one beam alone.

Can also write $\cos^2\left(\frac{k\delta}{2}\right) = \frac{1}{2}\left(1+\cos(k\delta)\right)$ or

$$E_p E_p^* = 2A^2\left(1+\cos k\delta\right)$$

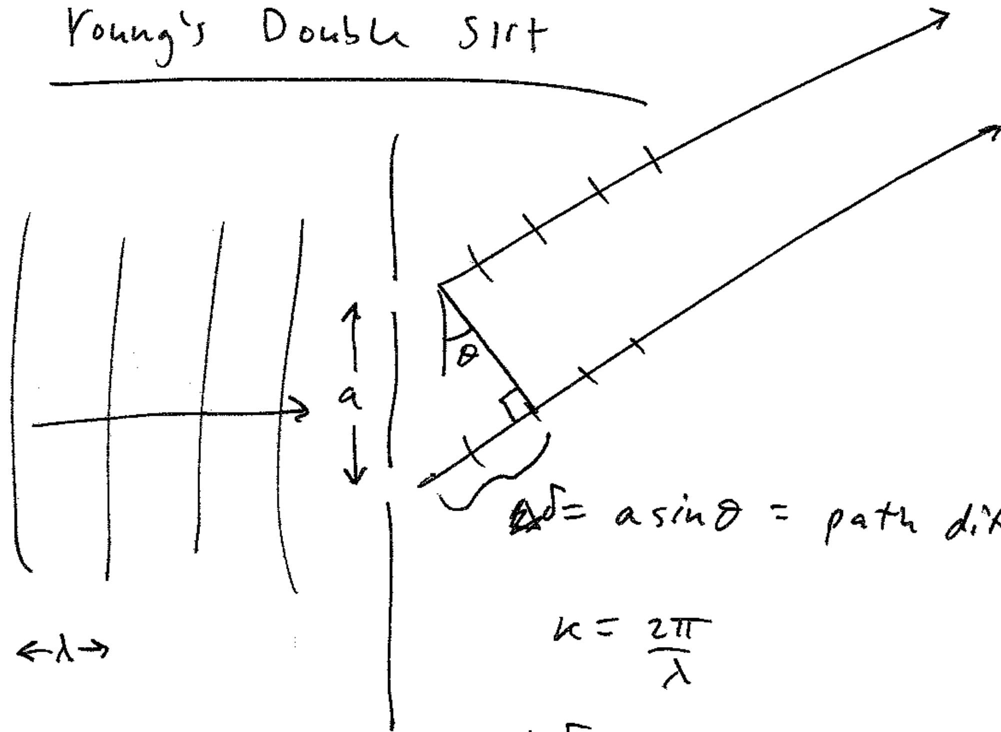
When $k\delta = m(2\pi)$, $m = 0, \pm 1, \pm 2, \ldots$

constructive interference

When $k\delta = (m+\frac{1}{2})(2\pi)$, $m = 0, \pm 1, \pm 2,$

destructive interference

Young's Double Slit



$\cancel{\Delta}\delta = a \sin\theta$ = path difference

$$k = \frac{2\pi}{\lambda}$$

$$k\delta = \frac{2\pi a \sin\theta}{\lambda}$$

$$E_p E_p^* = 4 \underset{\uparrow A^2}{I_0} \cos^2 \frac{k\delta}{2} = 4 I_0 \cos^2\left(\frac{\pi a \sin\theta}{\lambda}\right)$$

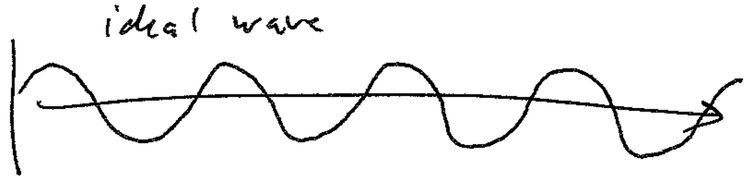
Coherence

Real light waves aren't perfect sines & cosines:

$$\cos(kx + \phi(t))$$

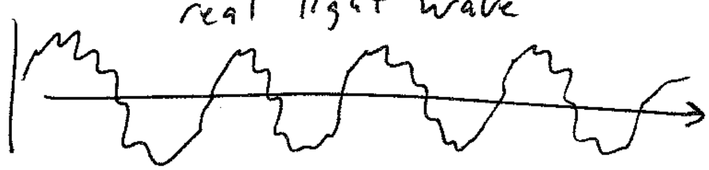
↑ some random function of time

ideal wave



perfectly predictable for ever

real light wave



only predictable for a time called the "coherence time"

Lasers → ↑ up to femtos.
incoherent → incandescent → less than 10^{-14} sec