One Dimensional Scalar Waves

$\psi(x,t) = A e^{i(kx-\omega t)}$

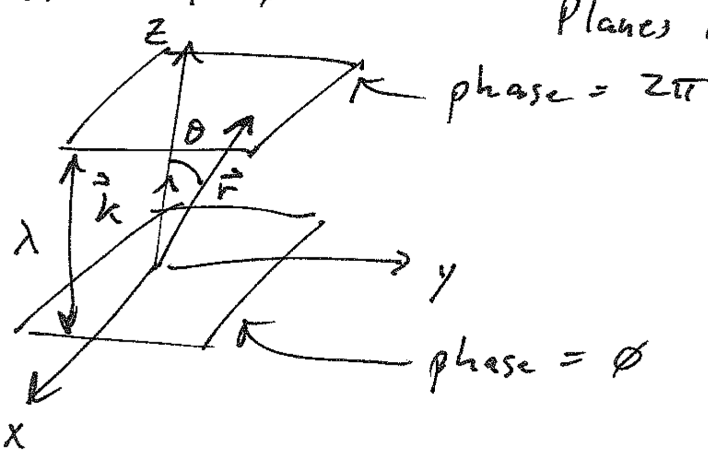
ψ ← scalar; A ← scalar; ← only depends on x, not (x,y,z)

Vector Wave in 3D

$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(???)}$

Snapshot at $t=\emptyset$:

Planes of constant phase



$\vec{k}$: in the direction of propagation, magnitude $|\vec{k}| = \frac{2\pi}{\lambda} = k$

$\vec{r}$: a vector to any point in space.

Suppose $\vec{r}$ points to a location in the plane with phase 2π:

Then $\vec{k}\cdot\vec{r} = k \underbrace{r\cos\theta}_{\lambda} = \frac{2\pi}{\lambda}\lambda = 2\pi$ ← correct phase

So at $t=\emptyset$:

$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r})}$

Now let time advance: phase $= \vec{k}\cdot\vec{r} - \omega t$:

$\vec{E}(\vec{r},t) = \vec{E}_0(\vec{k}\cdot\vec{r} - \omega t)$

EM waves have magnetic component:

$$\vec{B}(\vec{r},t) = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

Maxwell requires:

- $\vec{E}$ & $\vec{B}$ always have the same phase
- $\vec{E}\cdot\vec{B} = 0$ ← perpendicular
- $\vec{E}\cdot\vec{k} = \vec{B}\cdot\vec{k} = 0$ ← perpendicular to direction of travel
- $|\vec{E}| = c|\vec{B}|$ in MKS units

Poynting Vector:

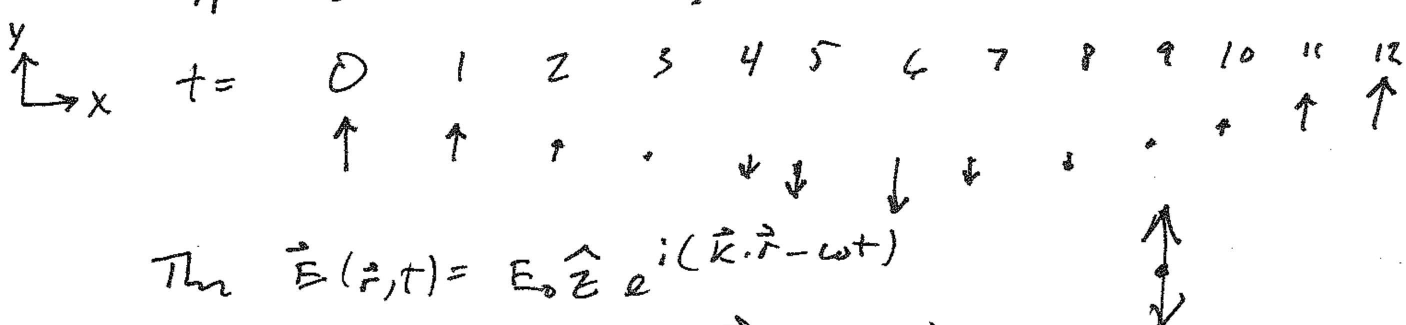
$$\vec{S} = \underbrace{\epsilon_0 c^2 \vec{E}\times\vec{B}}_{\text{power per unit area } \left(\frac{W}{m^2} \text{ in MKS}\right)}$$ points in direction of travel

$$\text{Irradiance} = I = \underbrace{\langle|\vec{S}|\rangle}_{\text{time average}} = \tfrac{1}{2}\epsilon_0 c E_0^2 = \tfrac{1}{2}\frac{c}{\mu_0}B_0^2$$

I ← measured by photodiode.

Linear Polarization

Suppose $\vec{E}$ oscillates along a line:



Then $\vec{E}(\vec{r},t) = E_0\hat{z}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$$I \propto E^2 = E_0^2 e^{i(2\vec{k}\cdot\vec{r} - 2\omega t)}$$

← irradiance varies twice as fast as $\vec{E}$.

Circular Polarization

Electric Field is constant in magnitude, but direction rotates:

t:	0	1	2	3	4	5	6	7	8
	↑	↖	←	↙	↓	↘	→	↗	↑

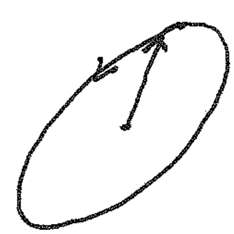
Circular Polarization is the sum of two linear polarization with <u>same magnitude</u> out of phase by $\frac{\pi}{2}$:

$$\vec{E}(\vec{r},t) = E_0 \hat{x}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)} + E_0 \hat{y}\, e^{i(\vec{k}\cdot\vec{r}-\omega t-\frac{\pi}{2})}$$

Elliptical Polarization

Same as circular except:

- magnitude of X and y polarizations differ
- AND/OR phase difference is not $\pi/2$



"Unpolarized" Light

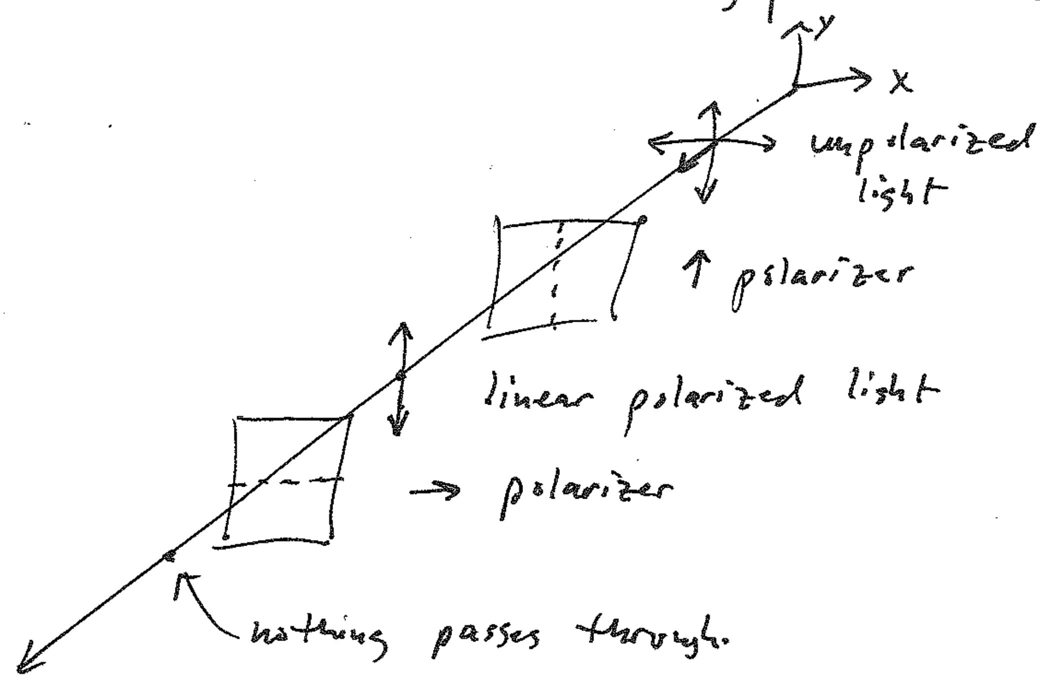
At any instant, $\vec{E}$ can only have one direction $\Rightarrow$ 100% polarized

But, if polarization direction changes randomly and quickly compared to the measurement time, we say the light is unpolarized.

- incoherent light sources like light bulbs are unpolarized.
- Lasers are often partially or fully polarized.

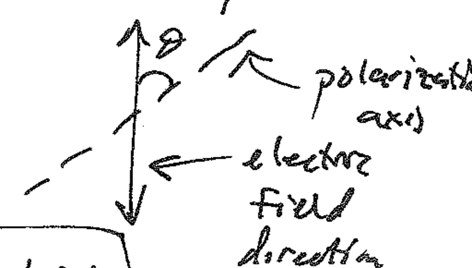
Dichroic "Linear" Polarizers

Absorbs electric field in one direction, passes it in the other



~~Suppose unpolarized light is incident on a Dichroic polarizer:~~

$$\cancel{I_{in} = I_x + I_y \propto E_x e^{2i(\vec{k}\cdot\vec{r}-\omega t)} + E_y e^{2i(\vec{k}\cdot\vec{r}-\omega t)}}$$



$$|\vec{E}|_{transmitted} = |\vec{E}_0|\cos\theta$$

$$I_{transmitted} \propto E_0^2\cos^2\theta$$

$$= \boxed{I_0\cos^2\theta \quad \text{Malus' Law}}$$

if incoming light is 100% linearly polarized

If incoming light is unpolarized,

$$I_{transmitted} = \frac{I_0}{2}\cos^2\theta$$

I_0 is unpolarized intensity.