Recap:

Standard dev = \( \sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N-1}} \), \( \bar{x} = \frac{\sum x_i}{N} \)

Standard dev of mean: \( \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N-1}} \)

Measurement: \( \bar{x} \pm \sigma_{\bar{x}} \)

Propagation of errors: if \( f = f(x, y, z, \ldots) \) then

\[
\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x} \sigma_x\right)^2 + \left(\frac{\partial f}{\partial y} \sigma_y\right)^2 + \left(\frac{\partial f}{\partial z} \sigma_z\right)^2}
\]

Complex wave:

\( \psi(x, t) = A e^{i(kx - \omega t)} \)

\( A \): amplitude
\( k \): wave number = \( \frac{2\pi}{\lambda} \), \( \lambda \): wavelength
\( \omega \): angular frequency = \( \frac{2\pi}{T} \), \( T \): period, \( \nu = \frac{1}{T} \)

\( \nu = \frac{c}{\lambda} = \lambda \nu \)

Geometric Optics: Ignore wave nature of light.
Assume light travels in straight lines called rays.
Speed of light in vacuum = \( c = 3.0 \times 10^8 \) m/s.
Light slows when travelling through matter:

index of refraction = \( n = \frac{c}{\nu} > 1 \)

\( n_{\text{vacuum}} = 1 \), \( n_{\text{water}} = 1.33 \), \( n_{\text{diamond}} = 2.417 \)

\( n_{\text{air}} = 1.00029 \), \( n_{\text{glass}} = 1.52 \)
Reflection and Refraction at interfaces

At an interface between two materials, some light reflects, and some refracts.

Refractive angle governed by Snell's Law:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

Reflection governed by Law of Reflection:

\[ \theta_1 = \theta_3 \]

Fermat's Principle of Least Time

Fermat (1601 - 1665) realized that Snell's Law and the Law of Reflection could be derived by assuming that light always travels on the path that takes the least time.

Snell's Law for Fermat's Principle

Light goes from A to C.

What is x?

Time of travel

\[ t = \frac{x}{v_1} + \frac{x}{v_2} = \frac{x}{n_1} + \frac{x}{n_2} \]

\[ d_1 = \sqrt{x^2 + x^2}, \quad d_2 = \sqrt{b^2 + (c-x)^2} \]
\[ c^2 = n_1 \sqrt{a^2 + x^2} + n_2 \sqrt{b^2 + (c-x)^2} \]

Find the \( x \) that minimizes \( c^2 \):

\[
\frac{dc^2}{dx} = \frac{1}{2} n_1 (2x) \frac{1}{\sqrt{a^2 + x^2}} + \frac{1}{2} n_2 (2(c-x))(c-x) \frac{1}{\sqrt{b^2 + (c-x)^2}} = 0
\]

\[
\frac{n_1}{\sqrt{a^2 + x^2}} \cdot x = \frac{n_2 (c-x)}{\sqrt{b^2 + (c-x)^2}}
\]

\[
\frac{n_1}{x} = \frac{n_2 (c-x)}{L_2}
\]

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2
\]

**Total Internal Reflection**

\[ n_2 > n_1 \]

If \( n_2 < n_1 \), then light bends away from the normal.

\[ n_1 > n_2 \quad \theta_2 > \theta_1 \]

If \( \theta_1 \) is large enough, then \( \theta_2 = 90^\circ \), and the refracted ray travels parallel to the surface.

\[ n_1 \sin \theta_1 = n_2 \sin 90^\circ = n_2 \]

\[ \sin \theta_1 = \frac{n_2}{n_1} \]

\( \theta_1 \), called the **refracted angle**, is
If $\theta_1 > \theta_c$, no light will be transmitted $\Rightarrow$ 100% reflection.

Total internal reflection only possible when $n_1 > n_2$, $\theta_1 > \theta_c$.

Optical Fibers

Take advantage of TIR to transmit light efficiently.

$n_r = \text{air}$

$n_g = \text{plastic, glass, or quartz } n = 1.5$

Then, $\theta_c = 42^\circ$.

Real fibers are coated with another plastic called cladding.
- Finger prints on bare fiber would change transmission.
- Scratches must allow light to leak out.

Cladding usually $n = 1.4$, then $\theta_c = 69^\circ$
Today's Lab

Ray emerges parallel but displaced by distance \( |d| \).

Homework: show that \( \tan(\beta) = \tan(\alpha) - \frac{|d|}{\cos \alpha} \).