Recap:

standard dev $= \sigma = \sqrt{\dfrac{\sum_i (x_i - \bar{x})^2}{N-1}}$, $\bar{x} = \dfrac{\sum_i x_i}{N}$

Stand dev of mean: $\dfrac{\sigma}{\sqrt{N-1}} = \sigma_{\bar{x}}$

Measurement: $\bar{x} \pm \sigma_{\bar{x}}$

Propagation of errors: if $f = f(x, y, z, \ldots)$ then

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial f}{\partial y}\sigma_y\right)^2 + \left(\frac{\partial f}{\partial z}\sigma_z\right)^2}$$

Complex wave:

$$\Psi(x,t) = A e^{i(kx - \omega t)}$$ ← remember to take the real part.

A = amplitude

k = wavenumber $= \frac{2\pi}{\lambda}$, λ = wavelength

ω = angular frequency $= \frac{2\pi}{T} = 2\pi\nu$, $\nu = \frac{1}{T}$

$v = \frac{\omega}{k} = \lambda\nu$

Geometric Optics: Ignore wave nature of light.
Assume light travels in straight lines called rays.
Speed of light in vacuum $= c = 3.0 \times 10^8$ m/s.
Light slows when travelling through matter:

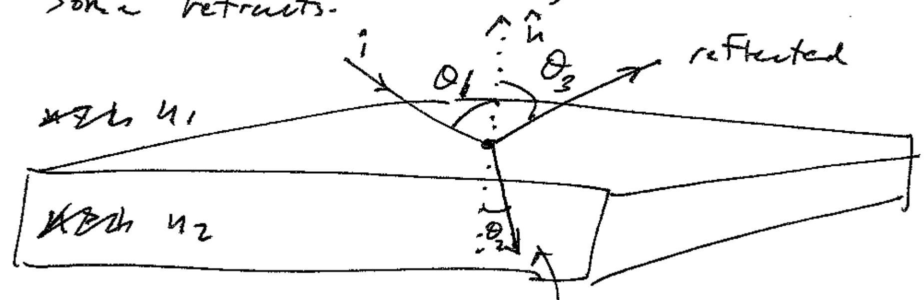
index of refraction $= n = \frac{c}{v} > 1$.

$n_{vacuum} = 1$, $n_{water} = 1.33$, $n_{diamond} = 2.417$

$n_{air} = 1.00029$, $n_{glass} = 1.52$

Reflection and Refraction at interfaces

At an interface between two materials, some light reflects, and some refracts.



Refraction angle governed by Snell's Law:

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

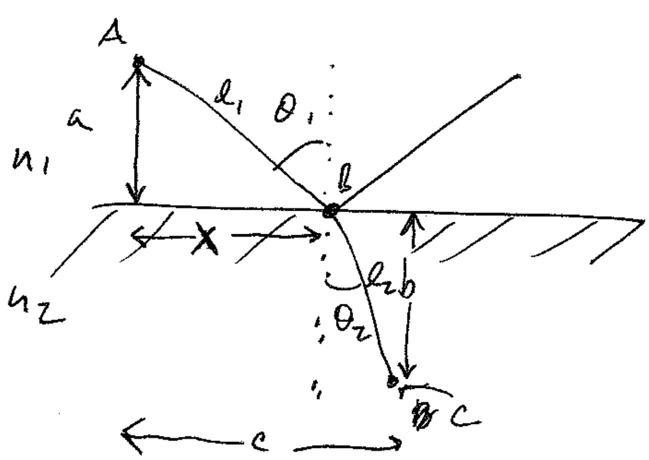
Reflection governed by Law of Reflection:

$$\boxed{\theta_1 = \theta_3}$$

Fermat's Principle of Least Time

Fermat (1601-1665) realized that Snell's Law and the Law of Reflection could be derived by assuming that light always travels on the path that takes the least time.

Snell's Law for Fermat's Principle



Light goes from A to C. What is x?

time of travel

$$t = \frac{l_1}{v_1} + \frac{l_2}{v_2} = \frac{l_1 n_1}{c} + \frac{l_2 n_2}{c}$$

$$l_1 = \sqrt{a^2 + x^2}\ , \quad l_2 = \sqrt{b^2 + (c-x)^2}$$

$$ct = n_1\sqrt{a^2+x^2} + n_2\sqrt{b^2+(c-x)^2}$$

Find the x that minimizes t:

$$\frac{dt}{dx} = \frac{\frac{1}{2}n_1(2x)}{\sqrt{a^2+x^2}} + \frac{\frac{1}{2}n_2\,(2(c-x))(-1)}{\sqrt{b^2+(c-x)^2}} = \phi$$

$$\frac{n_1 x}{\sqrt{a^2+x^2}} = \frac{n_2(c-x)}{\sqrt{b^2+(c-x)^2}}$$

$$n_1\frac{x}{l_1} = \frac{n_2(c-x)}{l_2}$$

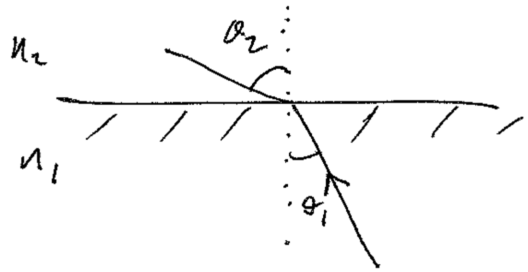
$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Total Internal Reflection

~~If $n_2 > n_1$)~~

If $n_2 < n_1$, then light bends away from the normal

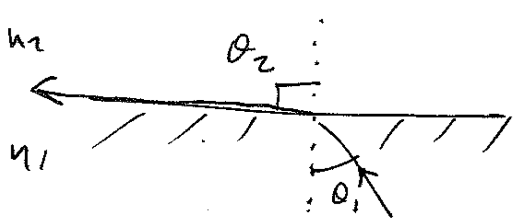
$n_1 > n_2$



$\theta_2 > \theta_1$

If θ_1 is large enough, then $\theta_2 = 90°$, and the refracted ray travels parallel to the surface

$n_1 > n_2$

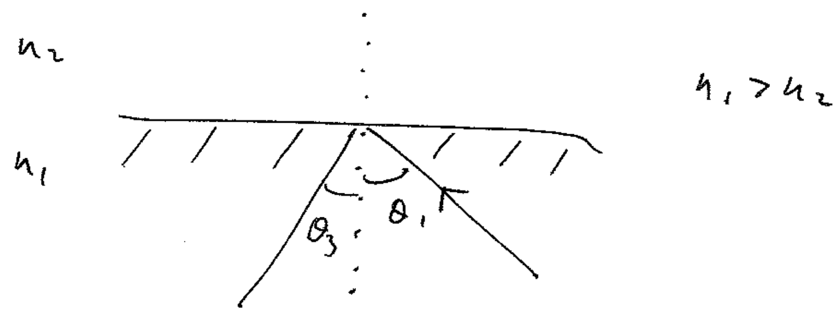


$n_1 \sin\theta_1 = n_2 \sin 90° = n_2$

$\sin\theta_1 = \frac{n_2}{n_1}$,

θ_1 called the critical angle

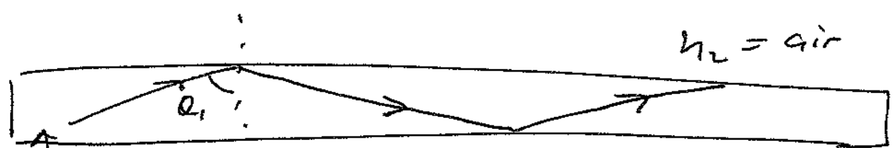
If $\theta_1 > \theta_c$, no light will be transmitted $\Rightarrow$ 100% reflection



Total internal reflection: only possible when $n_1 > n_2$, $\theta_1 > \theta_c$.

Optical Fibers

Take advantage of TIR to transmit light efficiently.



$\theta_1 > \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

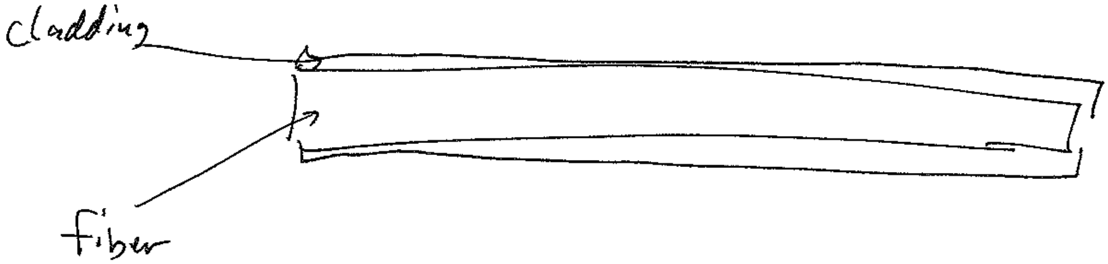
$n_1 =$ plastic, glass, or quartz $= n = 1.5$

Then $\theta_c = 42°$.

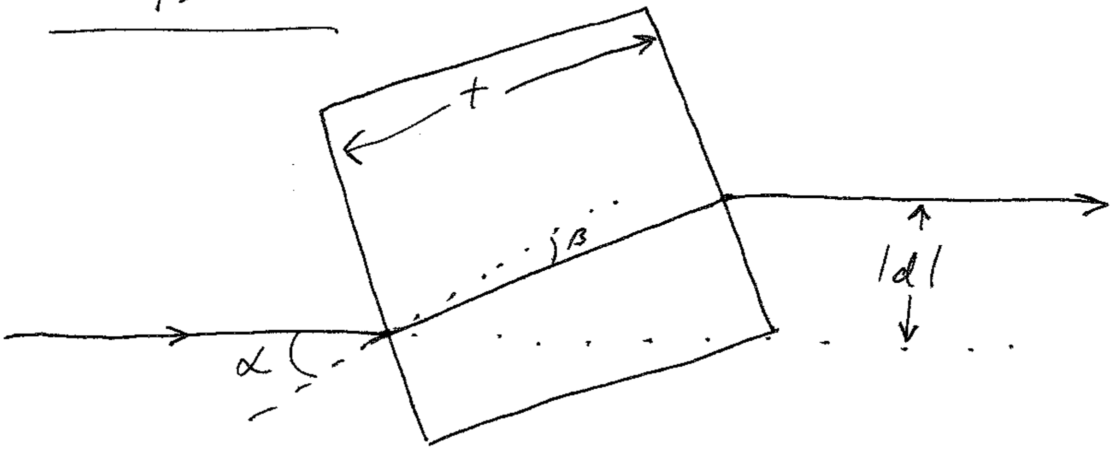
Real fibers are coated with another plastic called cladding

- Finger prints on bare fiber would change transmission
- scratches might allow light to leak out.

Cladding usually $n = 1.4$, then $\theta_c = 69°$



Today's Lab



Ray emerges parallel but displaced by distance $|d|$.

Homework: show that $\boxed{\tan(\beta) = \tan(\alpha) - \frac{|d|}{t\cos\alpha}}$