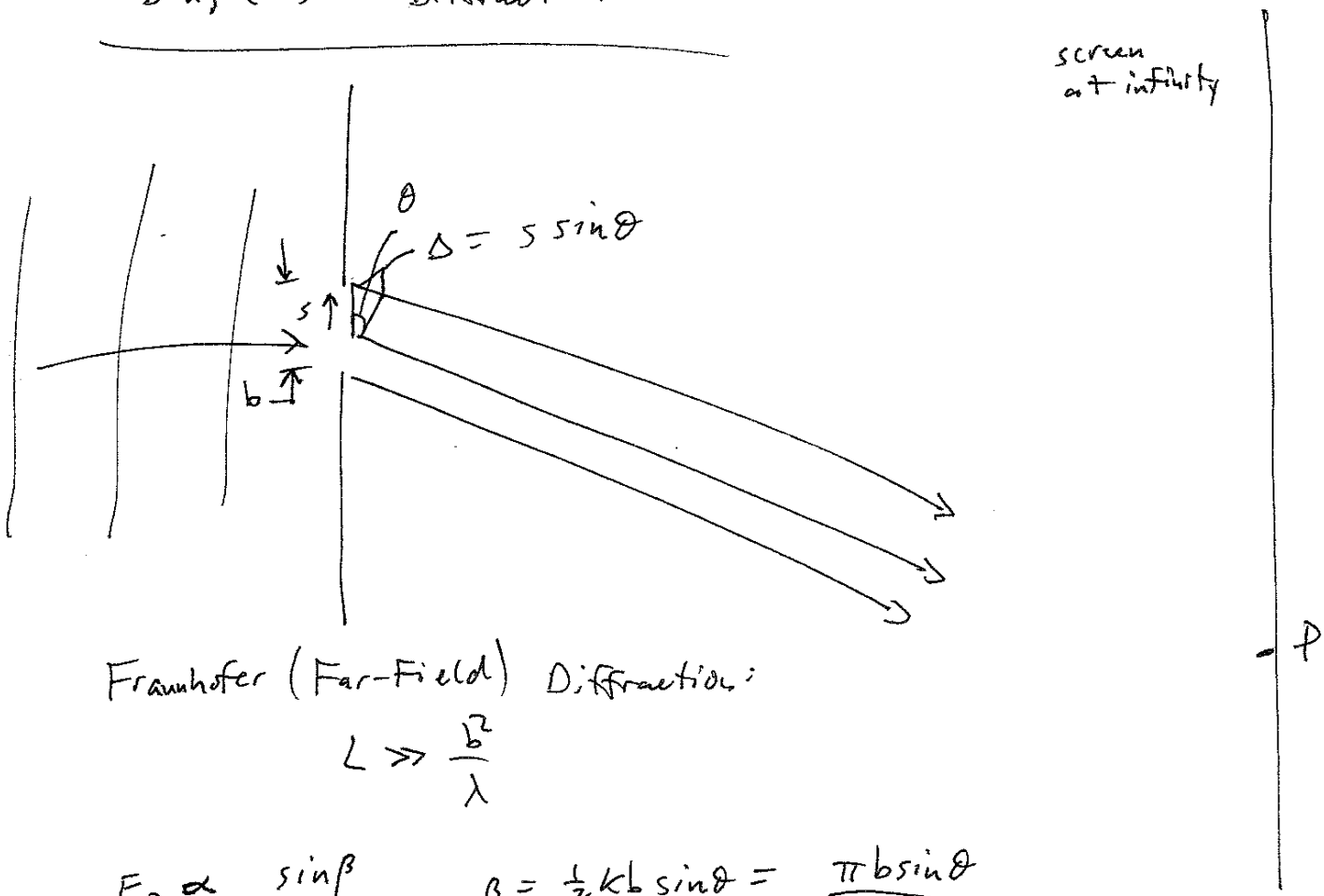
Lecture 10 - Double and Multiple Slit Diffraction 11/6/07

Single Slit Diffraction

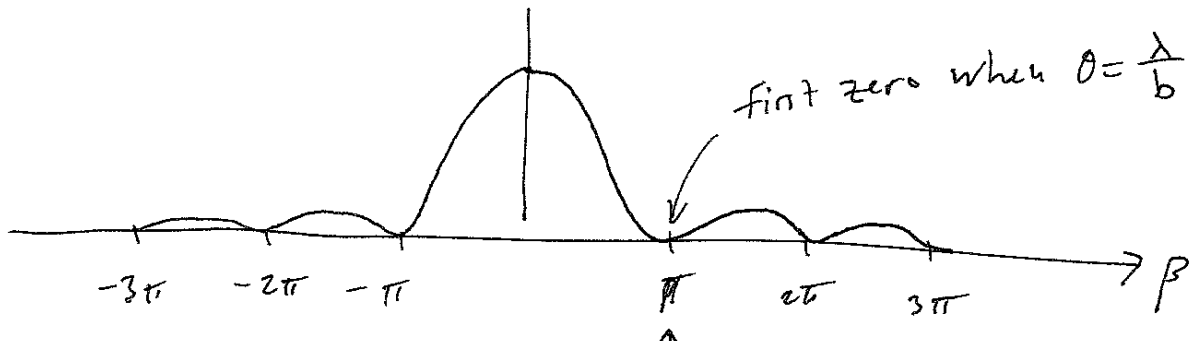


Fraunhofer (Far-Field) Diffraction:

$$L \gg \frac{b^2}{\lambda}$$

$$E_P \propto \frac{\sin\beta}{\beta}, \quad \beta = \tfrac{1}{2} k b \sin\theta = \frac{\pi b \sin\theta}{\lambda}$$

$$I = I_0 \frac{\sin^2\beta}{\beta^2}, \quad I = 0 \text{ when } \beta = m\pi, \; m = \pm 1, \pm 2, \pm 3, \text{ but not } m = 0!$$

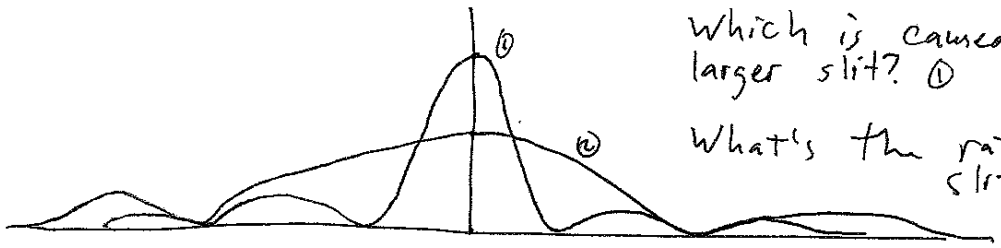


First zero when

$$\beta = \pi = \frac{\pi b \sin\theta}{\lambda}$$

$$\sin\theta = \lambda/b \approx \theta$$

Compare Two Diffraction Patterns:

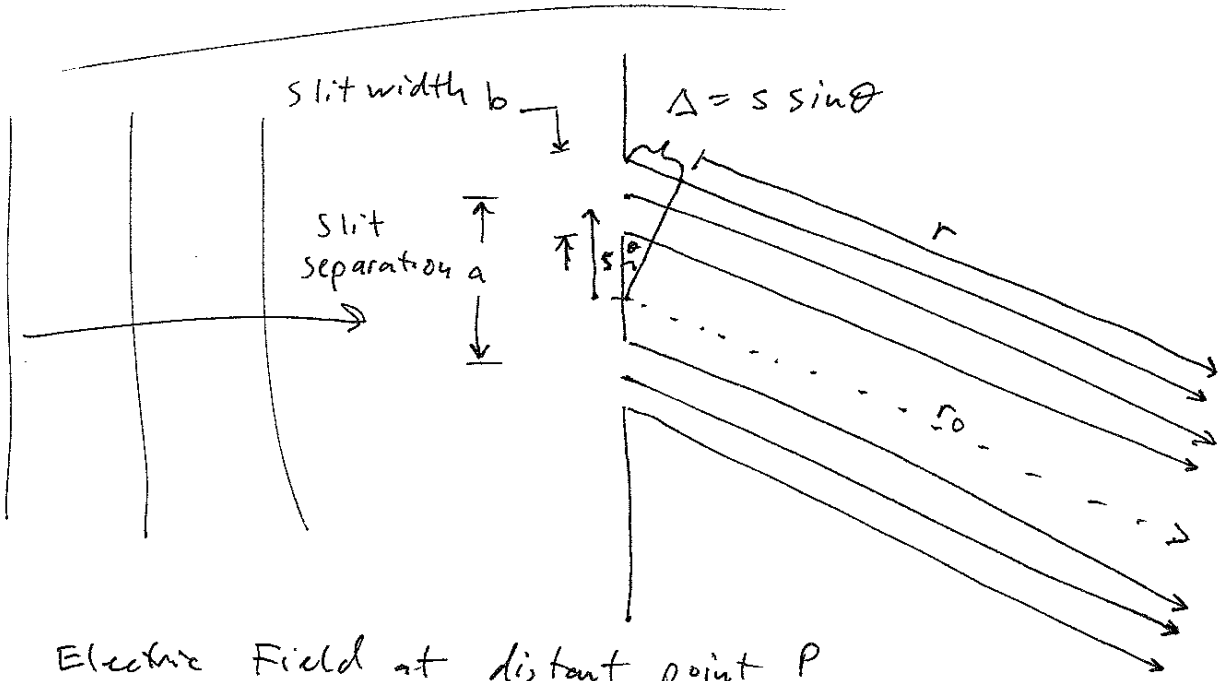


Which is caused by the larger slit? ①

What's the ratio of the slit widths? 2:1

The more you attempt to squeeze light (by putting it through narrower slits) the more it spreads out.

Double Slit Diffraction



Electric Field at distant point P due to small section of the slot at position s:

$$dE_P = \underbrace{\frac{E_L\, ds}{r}}_{\text{amplitude factor for spherical waves}} \underbrace{e^{i(kr-\omega t)}}_{\text{wave factor}} \qquad r = r_0 + \Delta = r_0 + s\sin\theta$$

Total Field at point P given by integrating over the entire aperture (integrating over s)

$$E_P = \int_{\text{Slits}} dE_P = \frac{E_L}{r_0+\Delta} \int_{\text{Slits}} e^{i(k(r_0+s\sin\theta)-\omega t)}\, ds$$

ignore Δ compared to r_0

$$= \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \int_{\text{Slits}} e^{iks\sin\theta}\, ds$$

$$= \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \left[\underbrace{\int_{-(a+b)/2}^{-(a-b)/2} e^{iks\sin\theta}\, ds}_{\text{lower slit}} + \underbrace{\int_{(a-b)/2}^{(a+b)/2} e^{iks\sin\theta}\, ds}_{\text{upper slit}} \right]$$

Homework: evaluate the integral.

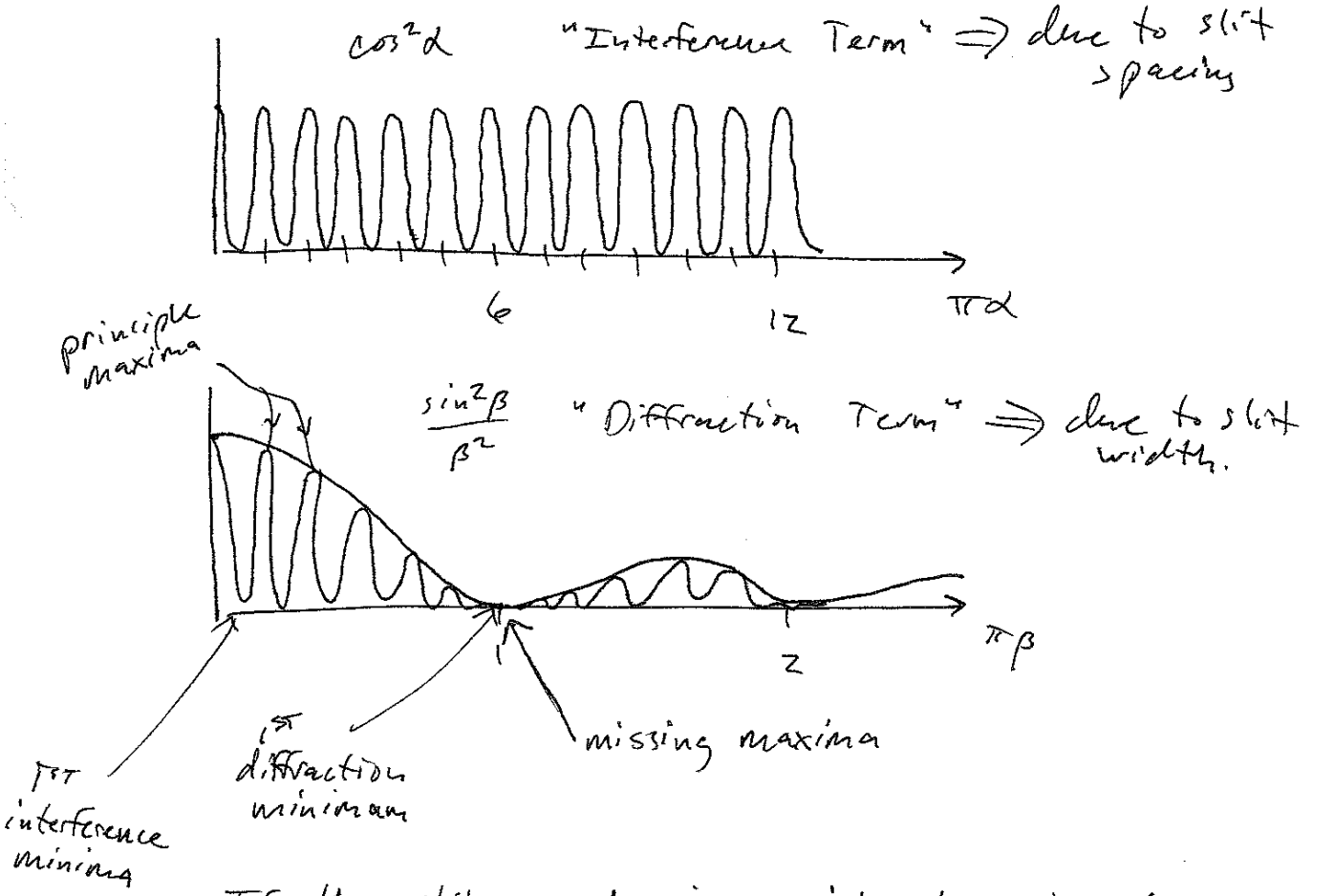
$$E_P = \frac{2E_L b}{r_0} e^{i(kr_0-\omega t)} \left(\frac{\sin\beta}{\beta}\right)(\cos\alpha)$$

$$\beta = \tfrac{1}{2}kb\sin\theta, \quad \alpha = \tfrac{1}{2}ka\sin\theta.$$

$$I \propto E_P E_P^* = \underbrace{\left(\frac{\sin^2\beta}{\beta^2}\right)}_{\text{"Diffraction Term" due to slit width}} \underbrace{(\cos^2\alpha)}_{\text{"Interference Term" Due to Having 2 Slits}}$$

Suppose $a = 6b \Rightarrow$ slit spacing is six times slit width:

Then $\alpha = 6\beta$



- If the slit spacing is an integral number of slit widths, we always have missing maxima
- Number of peaks in the central diffraction fringe $= 2\left(\frac{a}{b}\right) - 1$

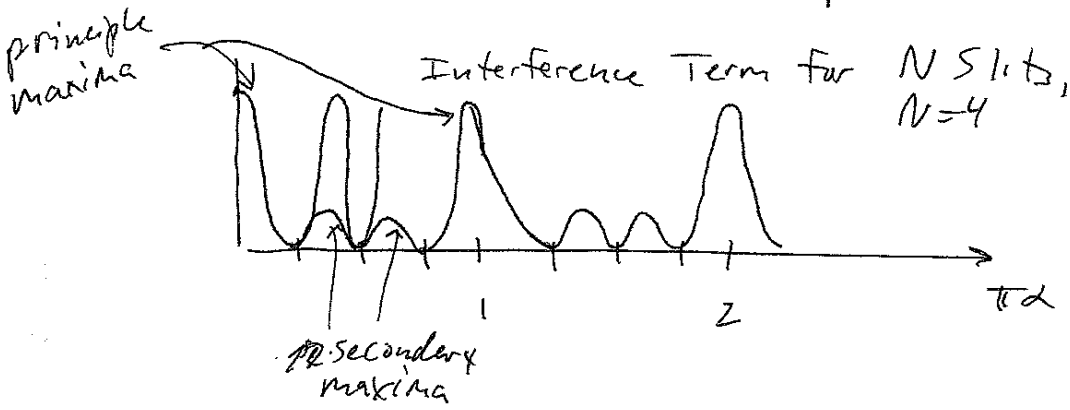
N Slits

$$I = I_0 \left(\frac{\sin^2\beta}{\beta^2}\right)\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

Interference Factor indeterminate whenever $\alpha = 0, \pi, 2\pi, \ldots$

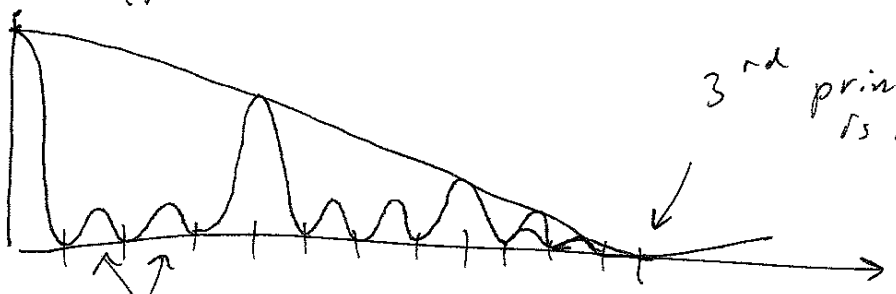
$$\lim_{\alpha \to m\pi} \frac{\sin N\alpha}{\sin\alpha} = \lim_{\alpha \to m\pi} \frac{N\cos N\alpha}{\cos\alpha} = N$$

∴ Maxima are N² more intense compared to a single slit diffraction pattern.



Number of Secondary maxima = N − 2
between principle maxima

Suppose



∴ a = 3b
slit spacing is 3 × slit width

two secondary maxima ⇒ Number of Slits = 4

When looking at an unfamiliar diffraction pattern

⇒ are there secondary maxima? If yes $N_{slits} > 2$

⇒ At what angle does the first diffraction minimum occur? Can it to determine slit widths

⇒ How many principle maxima come before the first diffraction minimum? Use to determine $\frac{a}{b}$, then get ab from measurement of b.