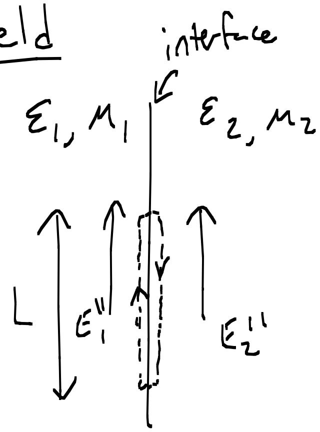
Boundary Conditions for Maxwell's eqns

E-field



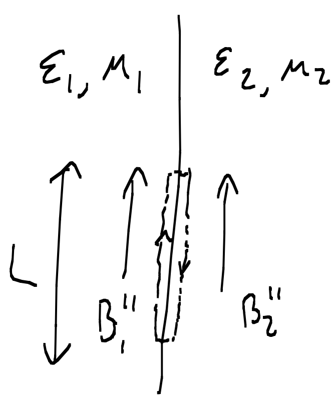
$$\oint \vec{E} \cdot d\vec{l} = -\cancel{\frac{d\Phi_B}{dt}} \rightarrow 0$$

"Faraday's Law"

$$E_1^{\parallel} L - E_2^{\parallel} L = 0$$

$$E_1^{\parallel} = E_2^{\parallel}$$

B-field



$$\oint \vec{B} \cdot d\vec{l} = \mu I + \mu \varepsilon \cancel{\frac{d\Phi_E}{dt}} \rightarrow 0$$

"Ampere's Law"

$$\oint \frac{\vec{B}}{\mu} \cdot d\vec{l} = 0$$

$$\frac{B_1^{\parallel}}{\mu_1} L - \frac{B_2^{\parallel}}{\mu_2} L = 0$$

$$\frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2} \qquad (H_1^{\parallel} = H_2^{\parallel})$$

Plane wave $\vec{B}$ Components from $\vec{E}$

$$\vec{E} = E_x e^{i(kz-\omega t)}\hat{x} \qquad (E_y = E_z = 0)$$

Faraday's Law: $\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ E_x e^{i(kz-\omega t)} & 0 & 0 \end{vmatrix} = ikE_x e^{i(kz-\omega t)}\hat{y}$$

$$\vec{B} = (B_x\hat{x} + B_y\hat{y} + B_z\hat{z})e^{i(kz-\omega t)}$$

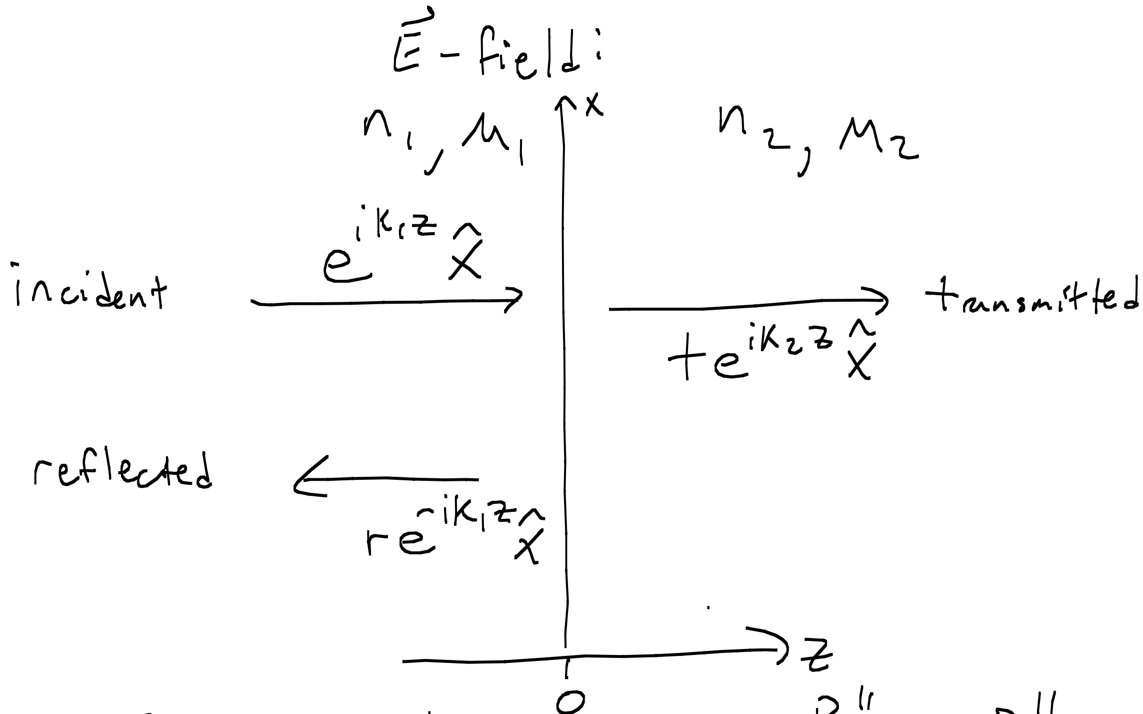
$$-\frac{d\vec{B}}{dt} = -\left(-i\omega(B_x\hat{x} + B_y\hat{y} + B_z\hat{z})e^{i(kz-\omega t)}\right)$$

So $\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$ implies $ikE_x e^{i(kz-\omega t)}\hat{y} = i\omega B_y e^{i(kz-\omega t)}\hat{y}$

Therefore, $B_y = \frac{k}{\omega}E_x = \frac{E_x}{v} = \frac{nE_x}{c}$ $\quad (B_x = B_z = 0)$

Note: $\vec{E}\times\vec{B}$ is propagation direction $(\hat{x}\times\hat{y} = \hat{z})$!

Imposing Boundary Conditions



B.C.'s: ① $E_1^{\parallel} = E_2^{\parallel}$ ② $\frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2}$ $\left(B^{\parallel} = \frac{n}{c}E^{\parallel}\right)$

①: $e^{ik_1 z} + re^{-ik_1 z}\Big|_{z=0} = te^{ik_2 z}\Big|_{z=0} \longrightarrow 1 + r = t$

$B^{\parallel} = \frac{E^{\parallel}}{v}$

②: $\frac{n_1}{\mu_1 c}e^{ik_1 z} - \frac{n_1}{\mu_1 c}re^{-ik_1 z}\Big|_{z=0} = \frac{n_2}{\mu_2 c}te^{ik_2 z}\Big|_{z=0} \longrightarrow \frac{n_1}{\mu_1} - r\frac{n_1}{\mu_1} = \frac{n_2}{\mu_2}t$

$$\frac{n_1}{\mu_1} - r\frac{n_1}{\mu_1} = \frac{n_2}{\mu_2}(1+r)$$

$$\frac{n_1}{\mu_1} = \frac{n_2}{\mu_2} + r\left(\frac{n_1}{\mu_1} + \frac{n_2}{\mu_2}\right)$$

$$r = \frac{\frac{n_1}{\mu_1} - \frac{n_2}{\mu_2}}{\frac{n_1}{\mu_1} + \frac{n_2}{\mu_2}}$$

Reflection Coefficient:

$$R = |r|^2 = \left(\frac{\frac{n_1}{\mu_1} - \frac{n_2}{\mu_2}}{\frac{n_1}{\mu_1} + \frac{n_2}{\mu_2}}\right)^2$$

When $\mu_1 = \mu_2$,

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

"transmission coef"

$$R + T = 1 \qquad T = 1 - R$$

$$T = 1 - \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \frac{(n_1+n_2)^2}{(n_1+n_2)^2} - \frac{(n_1-n_2)^2}{(n_1+n_2)^2}$$

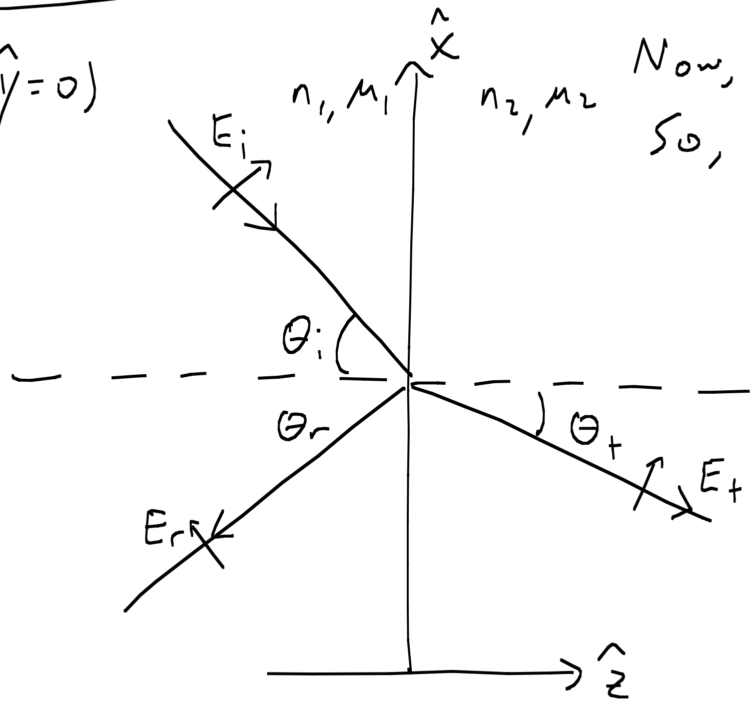
$$= \frac{n_1^2 + n_2^2 + 2n_1n_2 - (n_1^2 + n_2^2 - 2n_1n_2)}{(n_1+n_2)^2}$$

$$= \frac{4n_1n_2}{(n_1+n_2)^2}$$

Reflection is consequence of field conservation at the interface!

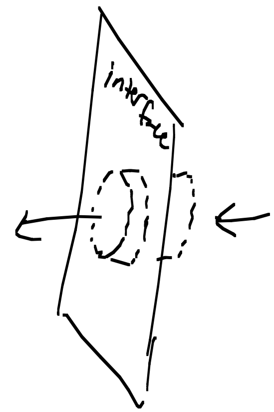
What about oblique incidence?

$(\vec{E}\cdot\hat{y}=0)$



Now, we have 4 unknowns $(E_r, \theta_r, E_t, \theta_t)$.

So, we need 2 more B.C.'s?:



$$\oiint \vec{B}\cdot d\vec{S} = 0$$ implies

(3) $B_1^\perp = B_2^\perp$

and

$$\oiint \vec{E}\cdot d\vec{S} = \frac{\cancel{Q}^{\,0}}{\varepsilon} = 0$$ no free charge

implies

(4) $\epsilon_1 E_1^\perp = \epsilon_2 E_2^\perp$