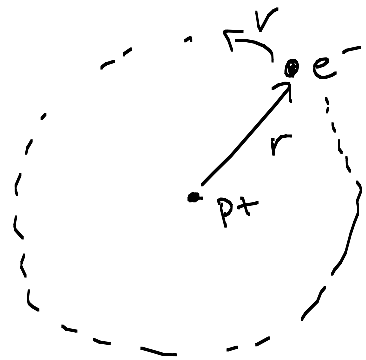
# Bohr model

"quantized" angular momentum

① $mvr = n\hbar$ $\qquad n = 1, 2, \ldots$
"principle quantum number"




② $\dfrac{mv^2}{r} = \dfrac{e^2}{r^2} \implies v = \sqrt{\dfrac{e^2}{mr}}$

[centripetal = Coulomb]

$$m\sqrt{\frac{e^2}{mr}}\, r = n\hbar \implies \sqrt{e^2 m r} = n\hbar \implies r = \frac{n^2\hbar^2}{e^2 m}$$

Total Energy = Kinetic + Potential $= \frac{1}{2}mv^2 - \frac{e^2}{r} = \frac{1}{2}\cancel{m}\frac{e^2}{\cancel{m}r} - \frac{e^2}{r}$

$$= -\frac{1}{2}\frac{e^2}{r}$$

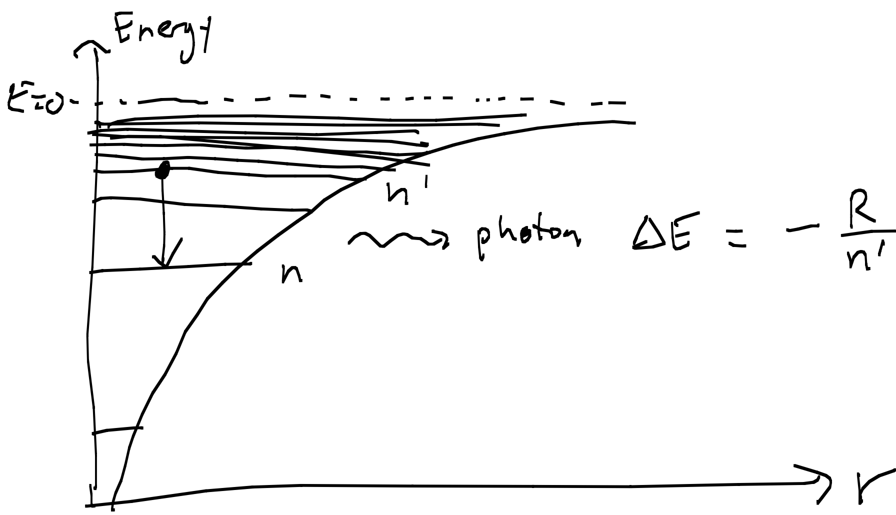
total energy $= -\frac{1}{2}\frac{e^2\, e^2 m}{n^2\hbar^2} = -\frac{1}{2}\left(\frac{e^2}{\hbar c}\right)^2 \frac{mc^2}{n^2}$

$\alpha \equiv \frac{e^2}{\hbar c}$ "fine structure constant" $\sim \frac{1}{137}$

$mc^2 \equiv E_0$ "rest mass energy" $\sim 511\,\text{KeV}$

$\text{Energy} = -\alpha^2 \frac{E_0}{2n^2}$

## "Radiative transitions"



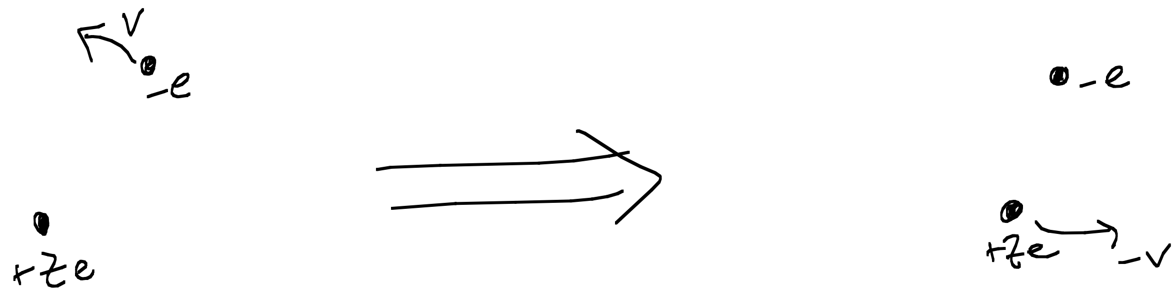

photon $\Delta E = -\frac{R}{n'^2} - \left(-\frac{R}{n^2}\right) = R\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$

Rydberg (1888)

$R \sim 13.6\,\text{eV}$

$E = h\nu \quad \lambda\nu = c, \ \nu = \frac{c}{\lambda} \Rightarrow E = \frac{\overbrace{hc}^{1240\,\text{nm}\cdot\text{eV}}}{\lambda}$

## Fine Structure: electron magnetic moment (spin)




nucleus at rest

electron at rest

$\vec{L} = -m\vec{v}\times\vec{r}$ $\qquad \mu_0 \varepsilon_0 = \frac{1}{c^2}$

magnetic field at electron due to "moving" nucleus (in electron's rest frame)

$$\vec{B} = \frac{\mu_0}{4\pi}\,\frac{Ze\,(-\vec{v}\times\vec{r})}{r^3} = \frac{1}{4\pi\varepsilon_0 c^2}\,\frac{Ze\vec{L}}{m r^3}$$

$$\text{Energy} = -\vec{\mu}\cdot\vec{B} = -\left(-g\frac{\mu_B}{\hbar}\vec{S}\right)\cdot\left(\frac{1}{4\pi\varepsilon_0 c^2}\,\frac{Ze\vec{L}}{mr^3}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{Ze^2}{2m^2c^2r^3}\,\vec{S}\cdot\vec{L}$$

"spin-orbit" $\qquad \mu_B = \frac{e\hbar}{2m}$

$$\langle E_{\text{spin-orbit}} \rangle \sim \frac{e^2}{m^2c^2}\frac{\hbar \cdot \hbar}{r^3} \qquad \langle r^3 \rangle \sim \left(\frac{\hbar^2}{e^2 m}\right)^3 = \frac{\hbar^6}{e^6 m^3}$$

$$\sim \frac{e^8 m^3 \hbar^2}{m^2 c^2 \hbar^6} = \frac{e^8 mc^2}{\hbar^4 c^2 c^2} = \left(\frac{e^2}{\hbar c}\right)^4 mc^2 = \alpha^4 mc^2$$

So spin-orbit is $\alpha^2$ smaller than gross electronic structure

for any fixed $n$, there are $2n-1$ possible values for $\vec{L}\cdot\hat{z}$ $(L_z)$ $\longrightarrow$ $E_{\text{spin-orbit}}$ is different, so otherwise degenerate states are split in energy.