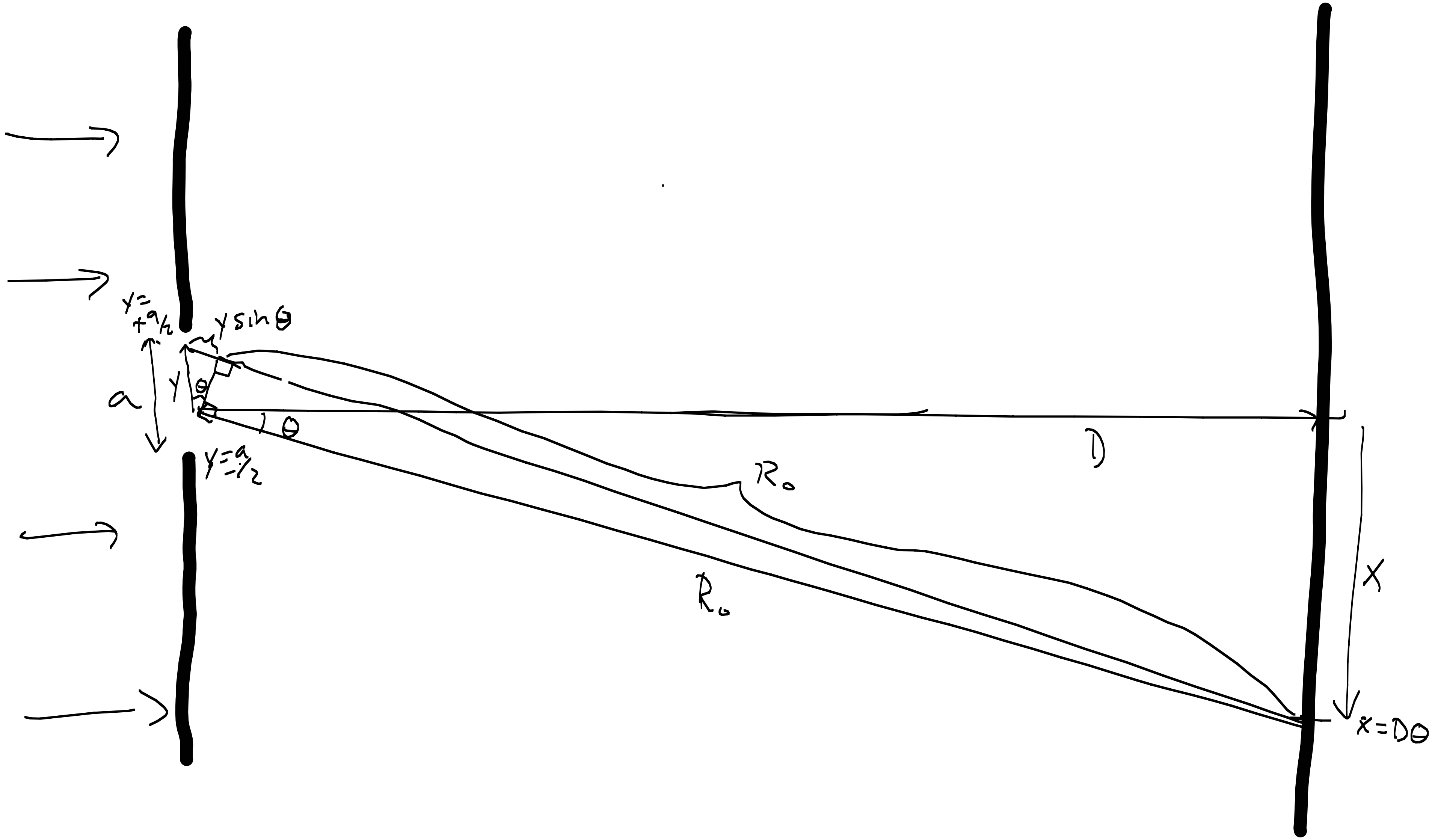
# Self-Interference: Diffraction

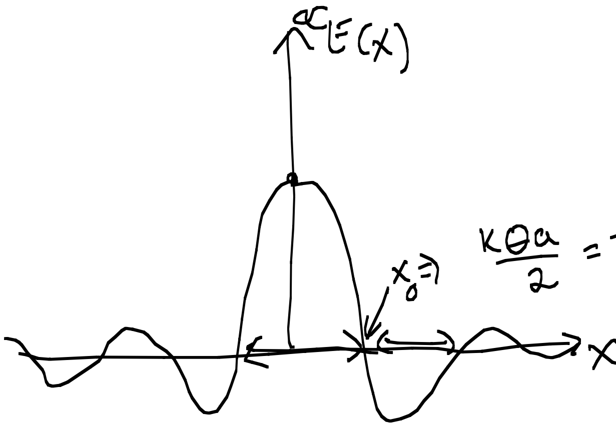


## E-field at $x = D\theta$

$$E \propto \int_{-a/2}^{+a/2} e^{ik(R_0 + y\sin\theta)}\,dy = e^{ikR_0}\int_{-a/2}^{+a/2} e^{iky\sin\theta}\,dy$$

$$= e^{ikR_0}\left.\frac{e^{iky\sin\theta}}{ik\sin\theta}\right|_{-a/2}^{+a/2} = e^{ikR_0}\,\frac{e^{ik\sin\theta\frac{a}{2}} - e^{-ik\sin\theta\frac{a}{2}}}{ik\sin\theta}$$

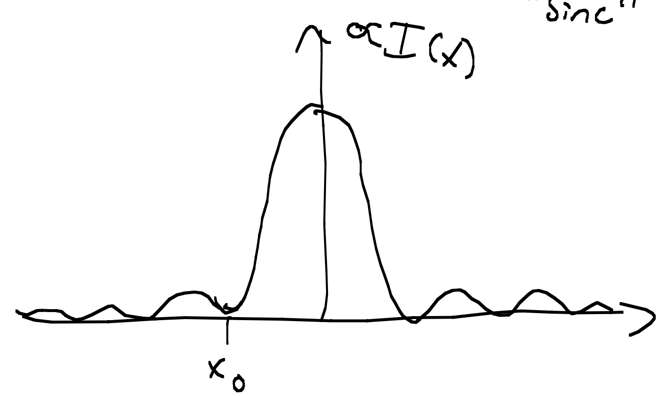
$$= \frac{2e^{ikR_0}\sin\left(k\sin\theta\frac{a}{2}\right)}{k\sin\theta} \underset{x \ll D}{\approx} \frac{\cancel{2}ae^{ikR_0}\sin\frac{k\theta a}{2}}{\cancel{2}\,\frac{k\theta a}{2}} = ae^{ikR_0}\underbrace{\frac{\sin\alpha}{\alpha}}_{\text{"sinc"}} \qquad \alpha \equiv \frac{k\theta a}{2}$$



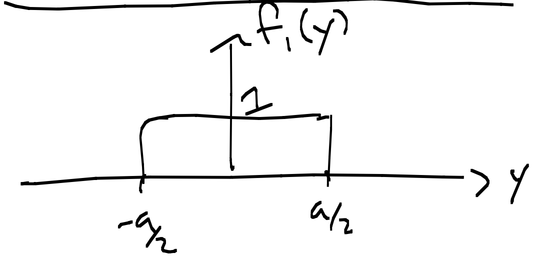

$$\frac{k\theta a}{2} = \pi$$

$$\frac{kxa}{2D} = \pi$$

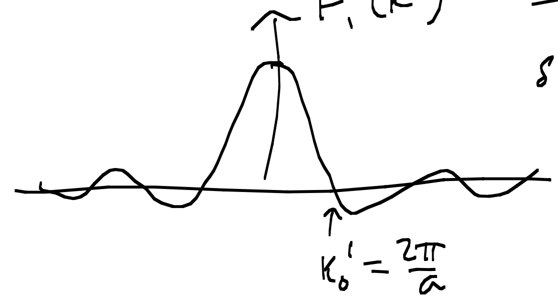
$$x_0 = \frac{2\pi D}{ka} = \frac{\lambda D}{a}$$



## Fourier Transform




$\xrightarrow[\text{Transform}]{\text{Fourier}}$




$$F_1(K') = \frac{\sin \frac{K'a}{2}}{K'a}$$

sinc!

To calculate diffraction pattern of an arbitrary aperture,

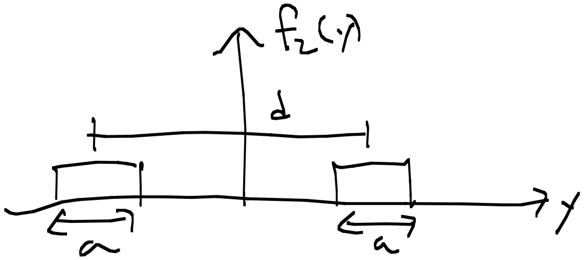
1. find $f(y)$ [aperture transmission fn.]
2. take Fourier Transform of $f(y)$
3. perform variable subs $K' \longrightarrow \frac{2\pi x}{D\lambda}$
4. find squared norm to get $I(x)$.

Example: (above)

$$\frac{\sin \frac{k'a}{2}}{k'a} \longrightarrow \frac{\sin \frac{2\pi x}{D\lambda}\frac{a}{2}}{\frac{2\pi x}{D\lambda}a} = \frac{\sin \frac{Kxa}{2D}}{\frac{Kxa}{D}} = \frac{\sin \frac{Ka\theta}{2}}{2K\theta a/2} = \frac{\sin\alpha}{2\alpha}$$

c.f. previous slide

# Example: Young's two-slit exp't

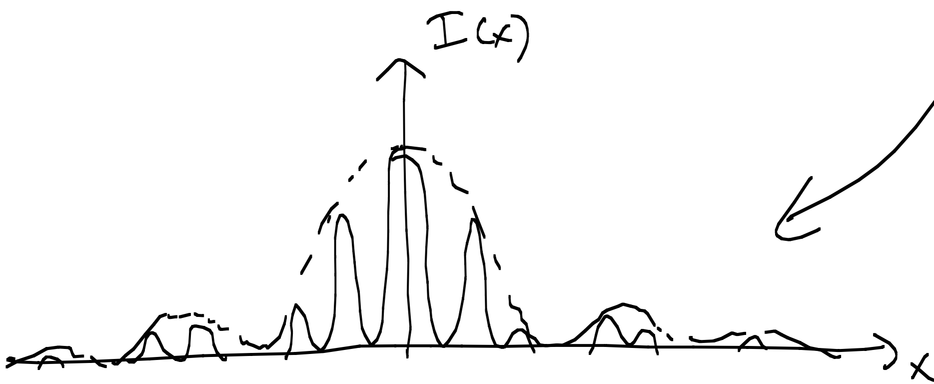



$$\text{F.T.}\left[f_2(y)\right] = \text{F.T.}\left[f_1\left(y+\tfrac{d}{2}\right) + f_1\left(y-\tfrac{d}{2}\right)\right]$$

$$= e^{-ik'\frac{d}{2}}\,\text{F.T.}\left[f_1(y)\right] + e^{+ik'\frac{d}{2}}\,\text{F.T.}\left[f_1(y)\right]$$

$$= \text{F.T.}\left[f_1(y)\right]\left(e^{ik'\frac{d}{2}} + e^{-ik'\frac{d}{2}}\right)$$

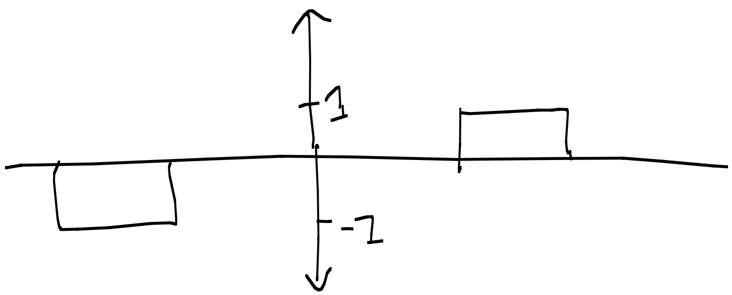
$$= \underbrace{2\cos\frac{k'd}{2}}_{\text{Interference of two apertures}}\ \underbrace{\text{F.T.}\left[f_1(y)\right]}_{\text{self-interference (diffraction)}}$$




$$\left(k' \to \frac{kx}{D}\right) \sim \cos\frac{kxd}{2D}$$

$I(x)$ modulated by $\cos\frac{kxd}{D}$

Example: $\Delta\phi = \pi$

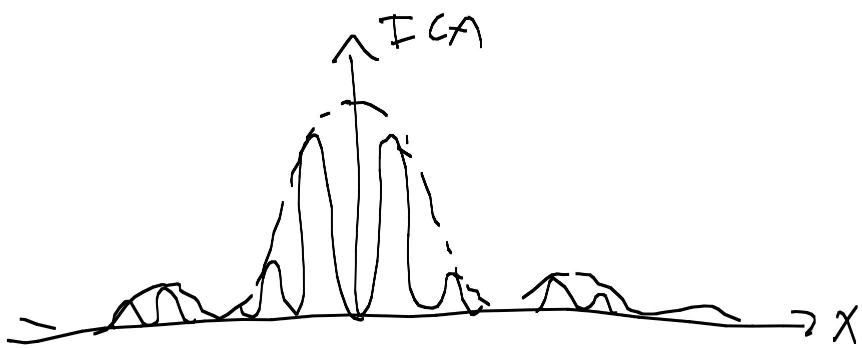



What is the diffracted image?

$$F.T.\left[f_1\left(y-\frac{d}{2}\right) - f_1\left(y+\frac{d}{2}\right)\right]$$

$$= \left(e^{ik'\frac{d}{2}} - e^{-ik'\frac{d}{2}}\right) F.T.\left[f_1(y)\right]$$

$$\left(2i\sin\frac{k'd}{2}\right) F.T.\left[f_1(y)\right]$$

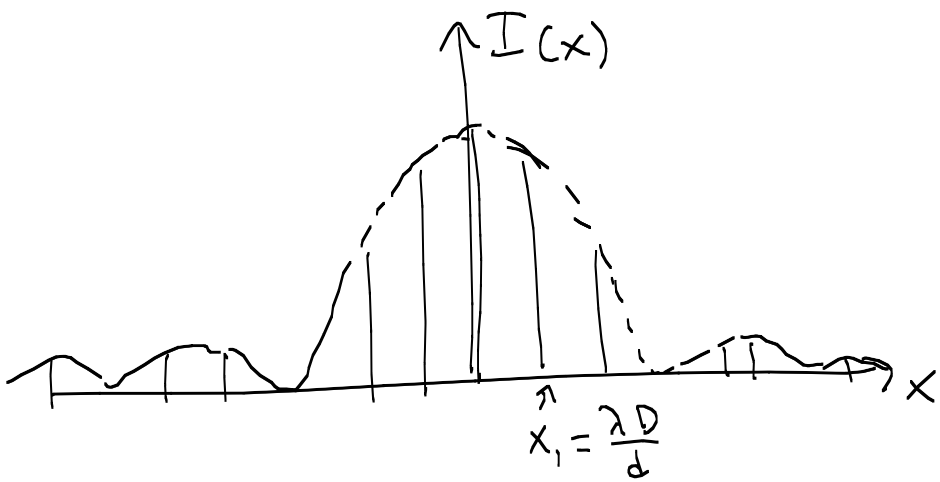


## Example: Infinite periodic symmetry


"diffraction grating"

$$F.T.\left[f_\infty(y)\right] = \sum_{n=-\infty}^{\infty} F.T.\left[f_1(y)\right]\,\delta\!\left(k' - n\frac{2\pi}{d}\right)$$

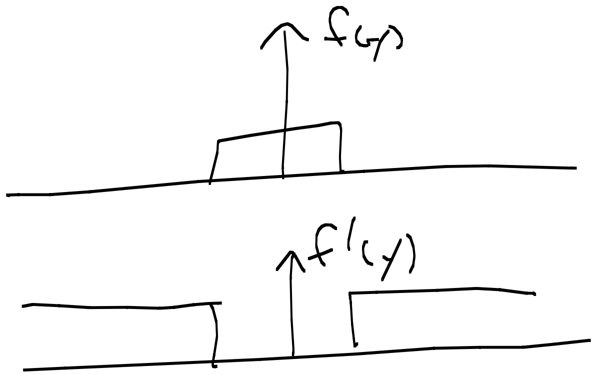
$$\delta\!\left(\frac{kx}{D} - n\frac{2\pi}{d}\right)$$




$$\frac{kx}{D} = \frac{2\pi}{d}$$

$$x = \frac{2\pi D}{dk} = \frac{\lambda D}{d}$$

## complementery apertures




$$f(y) = 1 - f'(y)$$

$$F.T.\left[f(y)\right] = F.T.\left[1\right] - F.T.\left[f'(y)\right]$$

$$= \delta(k') - F.T.\left[f'(y)\right]$$

$$E(x) = \delta(x) - E'(x)$$

For $\theta, x \neq 0$,

$$E(x) = -E'(x)$$

$$I(x) = I'(x)$$ "Babinet's principle"