Integral Form of Maxwell's Equations

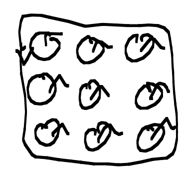
$$\oiint \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$ "Gauss' Law"

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$ "Faraday's Law"

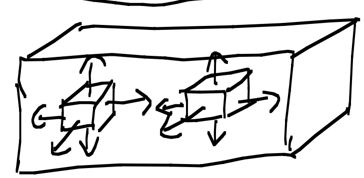
$$\oiint \vec{B} \cdot d\vec{S} = 0$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ "Ampere's Law"

"Stokes' Thm" $$\oint \vec{A} \cdot d\vec{l} = \iint \vec{\nabla} \times \vec{A} \cdot d\vec{S}$$



"divergence Thm" $$\oiint \vec{A} \cdot d\vec{S} = \iiint \vec{\nabla} \cdot \vec{A}\, dV$$



Maxwell's Eqns in Differential form

$$\oiint \vec{E}\cdot d\vec{S} = \iiint \vec{\nabla}\cdot\vec{E}\,dV = \frac{Q}{\varepsilon_0} = \frac{\iiint \rho\,dV}{\varepsilon_0} \Rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}}$$

$$\oint \vec{E}\cdot d\vec{l} = \iint \vec{\nabla}\times\vec{E}\cdot d\vec{S} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint \vec{B}\cdot d\vec{S} \Rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}}$$

$$\oiint \vec{B}\cdot d\vec{S} = \iiint \vec{\nabla}\cdot\vec{B}\,dV = 0 \Rightarrow \boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$$\oint \vec{B}\cdot d\vec{l} = \iint \vec{\nabla}\times\vec{B}\cdot d\vec{S} = \mu_0 I + \mu_0\varepsilon_0\frac{d\Phi_E}{dt} = \iint\left[\mu_0\vec{J} + \mu_0\varepsilon_0\frac{d\vec{E}}{dt}\right]\cdot d\vec{S}$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{d\vec{E}}{dt}}$$

Wave Equation

$\vec{\nabla}\times$ (Faraday's Law):

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}\times\left(-\frac{d\vec{B}}{dt}\right) = -\frac{d}{dt}\left(\vec{\nabla}\times\vec{B}\right) = -\frac{d}{dt}\left(\cancel{\mu_0\vec{J}}\xrightarrow{}0 + \mu_0\varepsilon_0\frac{d\vec{E}}{dt}\right)$$

in free space

$$\vec{\nabla}\left(\cancel{\vec{\nabla}\cdot\vec{E}}\right)\xrightarrow{}0 - \nabla^2\vec{E} = -\mu_0\varepsilon_0\frac{d^2\vec{E}}{dt^2}$$

in free space

$$\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{d^2\vec{E}}{dt^2}$$

$$\mu_0\varepsilon_0 \equiv \frac{1}{c^2} \qquad \left[\mu_0\varepsilon_0\right] = \frac{1}{\text{Velocity}^2}$$

$$\nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2\vec{E}}{dt^2} \qquad \text{"Wave equation"}$$

Note: in materials w/ $\varepsilon \neq \varepsilon_0$ and/or $\mu \neq \mu_0$, Velocity $= \frac{c}{n}$ where n is "index of refraction"

1-D $\left(\nabla^2 \to \frac{\partial^2}{\partial z^2}\right)$ solution

<u>Any</u> $F(z \pm ct)$! "D'Alembert" solution

"Simple" example in 1-D:

$$E = E_0 e^{i(kz \pm \omega t)} = E_0 e^{ik(z \pm \frac{\omega}{k} t)}$$ "plane wave"

$\frac{\omega}{k} = \frac{c}{n}$ $\quad$ $k \to$ "wavenumber" $\quad$ $k = \frac{2\pi}{\lambda}$ $\leftarrow$ "wavelength"