

Superposition of waves of different $|k|$

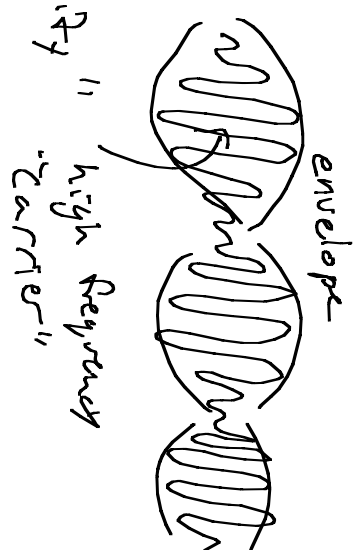
$$\frac{\partial^2}{\partial z^2} \vec{E} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} \quad \text{D'Alembert solution} \quad E(z \pm ct)$$

$c \rightarrow$ phase velocity $\frac{\omega}{k}$

$$\vec{E} = \cos\left[\left(k_0 - \frac{\Delta k}{2}\right)z - \left(\omega_0 - \frac{\Delta\omega}{2}\right)t\right] + \cos\left[\left(k_0 + \frac{\Delta k}{2}\right)z - \left(\omega_0 + \frac{\Delta\omega}{2}\right)t\right]$$

$$\cos(u-v) + \cos(u+v) = 2 \sin u \sin v$$

$$= 2 \underbrace{\sin(k_0 z - \omega_0 t)}_{\frac{\omega_0}{k_0} \text{ "phase velocity" }} \underbrace{\sin(\Delta k z - \Delta\omega t)}_{\frac{\Delta\omega}{\Delta k} \text{ "group velocity"}}$$



Continuum of waves: $\vec{E} \rightarrow \int_{-\infty}^{\infty} A(k) e^{i(kz - \omega(k)t)} dk$

for light in vacuum,
 $\omega(k) = ck$ dispers Ton rel'n,
 $\frac{\omega(k)}{k} = c$ and $\frac{d\omega(k)}{dk} = c$

$$\frac{\Delta\omega}{\Delta k} \rightarrow \frac{d\omega}{dk}$$

Dr. Bongo's Dispersion Lab

Get Help!

RESET ALL

Dispersion Relation: $\omega(k)$

0.01*k*(1.4)

Pulse Width (w)

0.1

Central Wavenumber (k)

75

Envelope: $f(x,w)$

$\exp(-(x/(w^2))^2)$

x-Axis Velocity

0.0

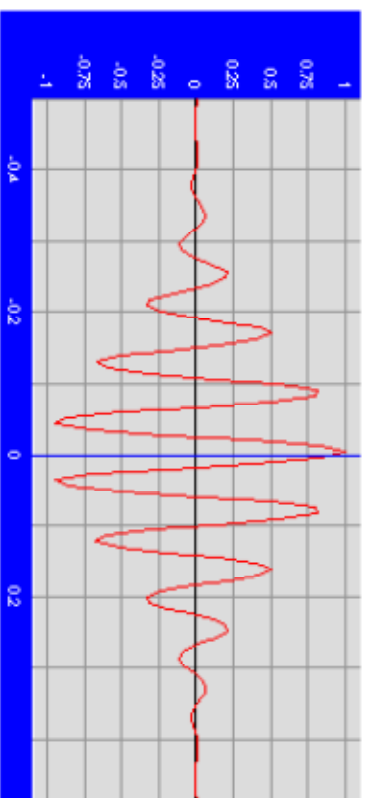
Time

0.0

Go!

Reset=0

The wave: $f(x)$

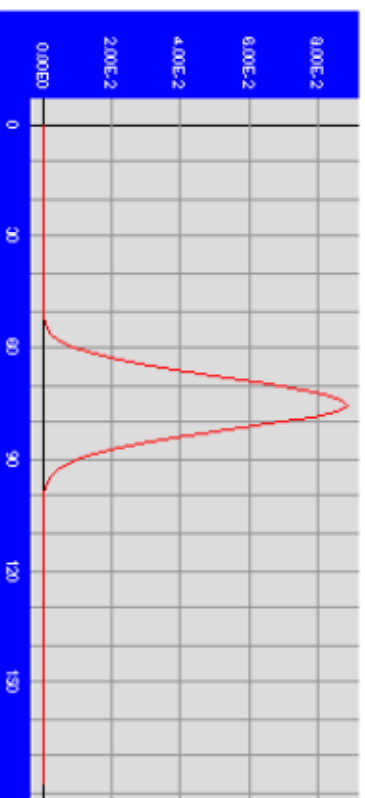


$\Delta x = 0.1$

$\Delta k = 5.01$

$\Delta x \Delta k = 0.5$

Its spectrum: $A(k)$

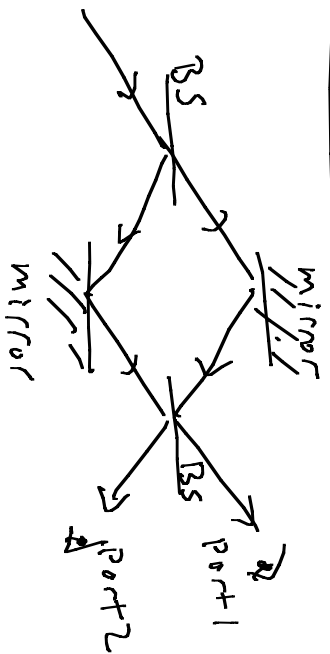


The simulation has gone as far as it can reliably go. Reset time to 0 to continue.

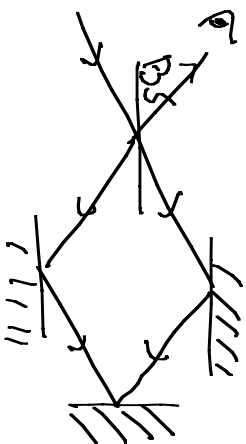
(c) Dailin S. Durfee

2004

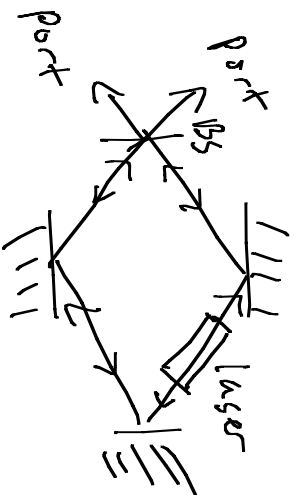
Other Interferometers:



"Mach-Zehnder"



"Sagnac"



"Ring laser"

Applications

Tests of Relativity:

Special

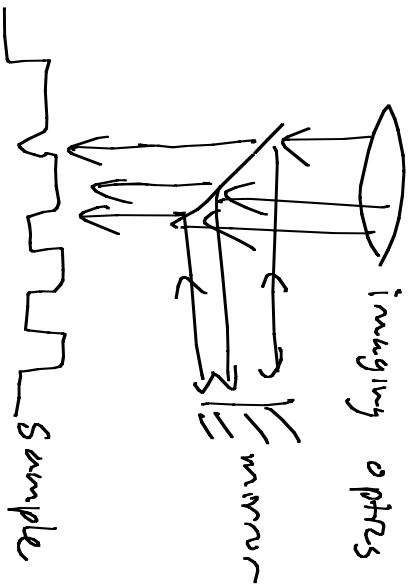
Michelson-Morley expt.

Fizeau

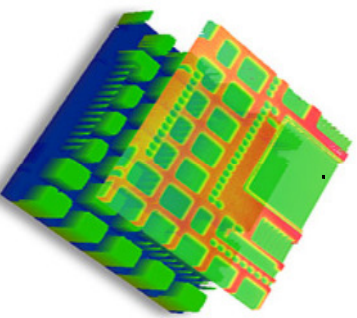
General

LIGO: 2 modified MM interferometers (LA + WA)
2.5 miles arm length.

Metology:



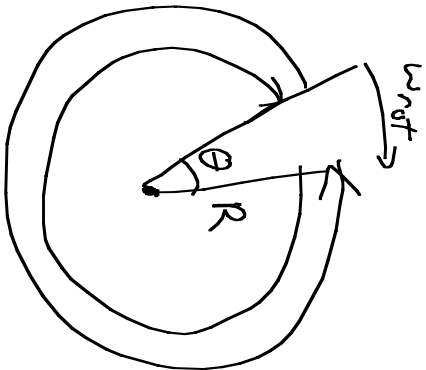
Imaging
interferometer



Zygo.com

Gyroscopes: inertial guidance

Sagnac interferometer:



$$t \approx \frac{2\pi R}{c} \pm \frac{(w_{rot})R}{c}$$

$t: c/w$
 $t: cw$

$$t \left(1 \mp \frac{w_{rot}R}{c}\right) \approx \frac{2\pi R}{c}$$

$$\boxed{(1 \pm \epsilon)^n \sim 1 \pm n\epsilon}$$

$\epsilon \ll 1$

$$t = \frac{2\pi R}{c} \left(1 \mp \frac{w_{rot}R}{c}\right)^{-1} \sim \frac{2\pi R}{c} \left(1 \pm \frac{w_{rot}R}{c}\right)$$

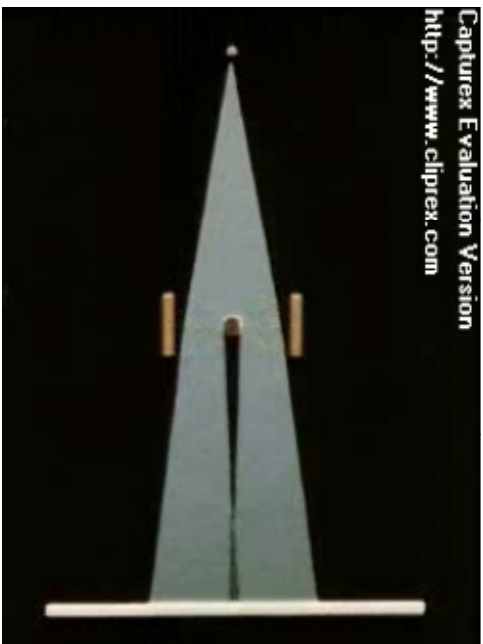
$$\Delta\phi = w \Delta t = 2w \frac{2\pi R}{c} \frac{w_{rot}R}{c} = \frac{4\pi R^2}{c^2} w w_{rot}$$

Interference

Important for airplane/submarine/etc navigation

Electron Interferometry: Vacuum

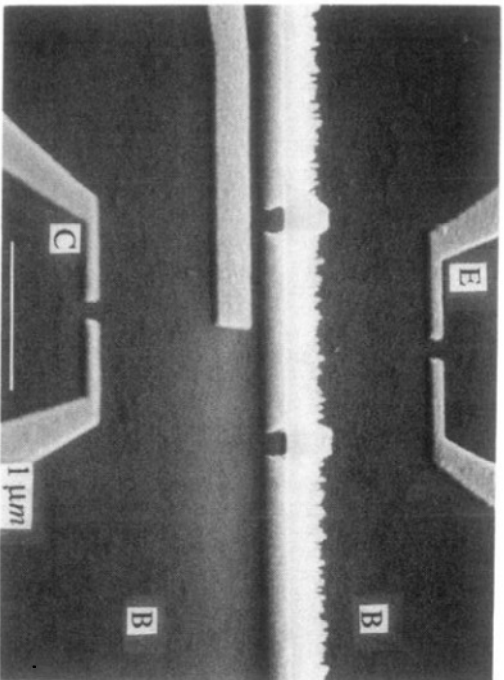
electrostatic biprism



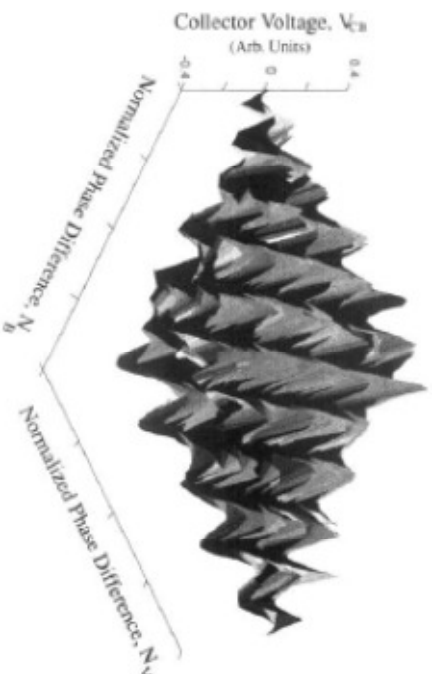
Like Young's 2-slit experiment



Electron Interferometry: Solid State
Two-slit:



A. Yacoby, M. Heiblum, V. Umansky, H. Shtrikman, and D. Mahalu
Phys. Rev. Lett. 73, 3149 (1994)



An electronic Mach-Zehnder interferometer

Yang Ji, Yunchul Chung, D. Sprinzak, M. Heiblum, D. Mahalu & Hadas Shtrikman

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