

Superposition of waves

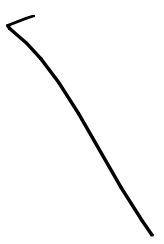
$$\nabla^2 \vec{E}_1 = \frac{1}{c^2} \frac{\partial^2 \vec{E}_1}{\partial t^2}$$

IF \vec{E}_2 is also a solution, $\nabla^2 \vec{E}_2 = \frac{1}{c^2} \frac{\partial^2 \vec{E}_2}{\partial t^2}$

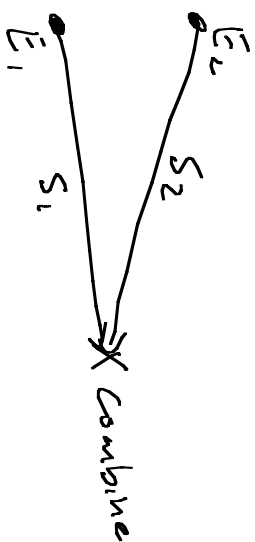
Then $\vec{E} = \vec{E}_1 + \vec{E}_2$ is also a solution:

$$\nabla^2 (\vec{E}_1 + \vec{E}_2) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} (\vec{E}_1 + \vec{E}_2)$$

$$\underbrace{\nabla^2 \vec{E}_1}_{=} + \underbrace{\nabla^2 \vec{E}_2}_{=} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \underbrace{\vec{E}_1}_{=} + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \underbrace{\vec{E}_2}_{=}$$



Superposition of 2 waves (Same k and polarization)



$$\vec{E}_1 = E_{01} \cos(\alpha_1 - \omega t)$$
$$\alpha_1 = kS_1 + \phi_1$$

$$\vec{E}_2 = E_{02} \cos(\alpha_2 - \omega t)$$
$$\alpha_2 = kS_2 + \phi_2$$

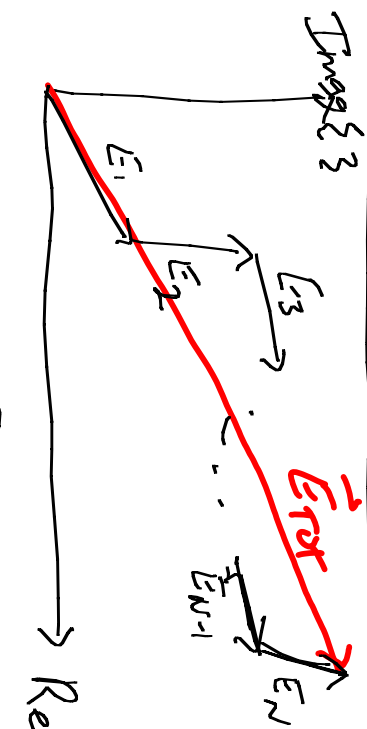
$$E_1 + E_2 = E_{01} \cos(\alpha_1 - \omega t) + E_{02} \cos(\alpha_2 - \omega t)$$
$$= E_{01} \cos(\alpha_1 - \omega t) + E_{01} \cos(\alpha_2 - \omega t) + (E_{02} - E_{01}) \cos(\alpha_2 - \omega t)$$

$$\cos u + \cos v = 2 \cos \frac{u+v}{2} \cos \frac{u-v}{2}$$

$$= 2 E_{01} \cos\left(\frac{\alpha_1 + \alpha_2 - 2\omega t}{2}\right) \cos\left(\frac{\alpha_1 - \alpha_2}{2}\right) + (E_{02} - E_{01}) \cos(\alpha_2 - \omega t)$$

$$\text{IF } \alpha_1 - \alpha_2 = 0, 2\pi, 4\pi, \dots - 2m\pi \quad m = 0, \pm 1, \pm 2, \dots \quad \text{"constructive"}$$
$$= \pi, 3\pi, \dots - (2m+1)\pi \quad \text{"destructive"}$$

Superposition of N waves: Phasors



$$\vec{E} \sim E e^{i(kx - \omega t)}$$

$$\vec{E}_{tot} = \left[\sum_i^N E_{0i} \cos \alpha_i \right] + i \left[\sum_i^N E_{0i} \sin \alpha_i \right]$$

$$(a + ib)^2$$

$$I = |\vec{E}_{tot}|^2 =$$

$$\left[\sum_i^N E_{0i} \cos \alpha_i \right]^2 + \left[\sum_i^N E_{0i} \sin \alpha_i \right]^2$$

$$= a^2 + b^2 + ab + ba$$

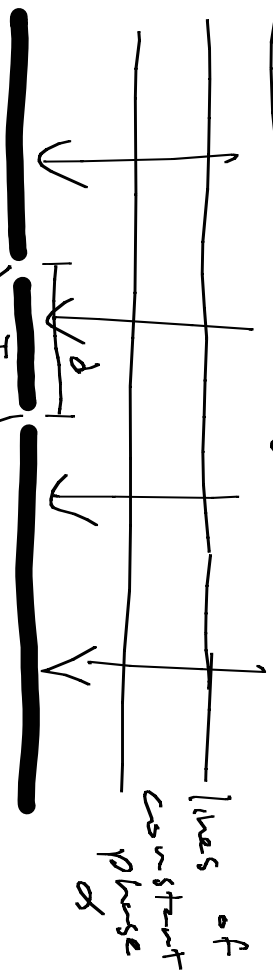
$$= \sum_{i=1}^N E_{0i}^2 \cos^2 \alpha_i + \sum_{i=1}^N E_{0i}^2 \sin^2 \alpha_i + 2 \sum_{j>i}^N \sum_{i=1}^N E_{0i} E_{0j} \cos \alpha_i \cos \alpha_j + 2 \sum_{j>i}^N \sum_{i=1}^N E_{0i} E_{0j} \sin \alpha_i \sin \alpha_j$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B, \quad \sin^2 \alpha + \cos^2 \alpha = 1$$

$$= \sum_{i=1}^N E_{0i}^2 + 2 \sum_{j>i}^N \sum_{i=1}^N E_{0i} E_{0j} \cos(\alpha_j - \alpha_i)$$

Second term \rightarrow D
when sources are incoherent!

Example! Young's two-slit experiment



lines of constant phase α

$$I(x) = \frac{I_0}{2} + \frac{I_0}{2} + 2\sqrt{\frac{I_0}{2}}\sqrt{\frac{I_0}{2}} \cos(\alpha_1 - \alpha_2)$$

$$= I_0 + I_0 \cos(\alpha_1 - \alpha_2)$$

$$\alpha_1 - \alpha_2 = k(s_1 - s_2) = k\left(\sqrt{L^2 + \left(x + \frac{d}{2}\right)^2} - \sqrt{L^2 + \left(x - \frac{d}{2}\right)^2}\right)$$

$$L \gg dx \Rightarrow kL \left(\sqrt{1 + \frac{\left(x + \frac{d}{2}\right)^2}{L^2}} - \sqrt{1 + \frac{\left(x - \frac{d}{2}\right)^2}{L^2}} \right)$$

$$\sim kL \left(1 + \frac{x^2 + xd + \frac{d^2}{4}}{2L^2} - \left(1 + \frac{x^2 - xd + \frac{d^2}{4}}{2L^2} \right) \right)$$

$$= \frac{kx d}{L}$$

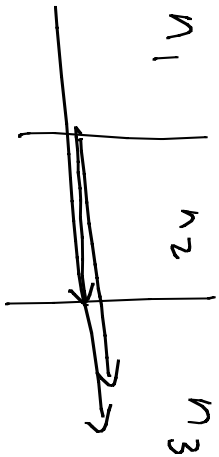
$$I(x) = I_0 \left(1 + \cos \frac{kx d}{L} \right)$$



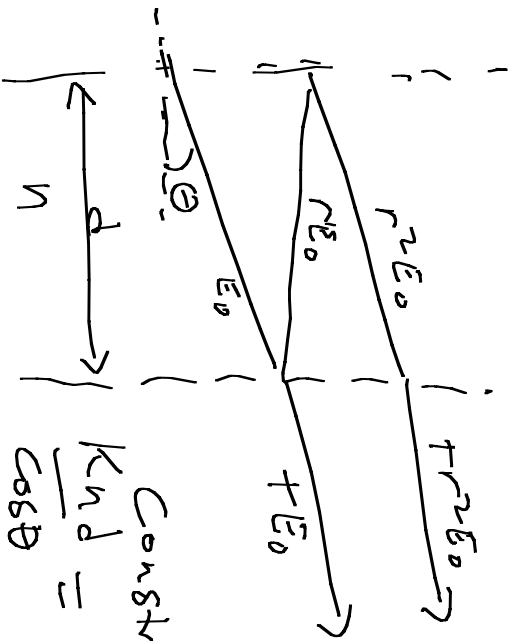
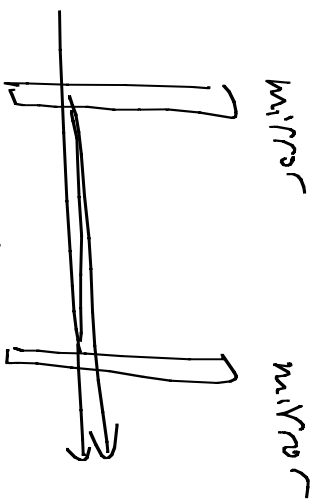
Bright spot when $\frac{kx d}{L} = 0, 2\pi, \dots, 2m\pi$ "constructive"

$$x = \frac{2m\pi L}{kd} = \frac{mL\lambda}{d}$$

Example: Fabry - Perot Etalon



OR



Constructive:

$$\frac{k_{nd}}{\cos\theta} = 0, \pi, 2\pi, \dots, m\pi$$

$$d = \frac{m\pi \cos\theta}{k_n} = \frac{\lambda_m \cos\theta}{2n}$$

destructive:

$$= \frac{\lambda_n}{2}, \frac{3\lambda_n}{2}, \dots$$

$$E = +E_0 \left[2 \cos \left(\frac{2\alpha_1 - 2k_{nd}}{2} - \omega t \right) \right]$$

$$\alpha_1 - \alpha_2 = 2 \frac{k_{nd}}{\cos\theta}$$

$$= \cos \frac{k_{nd}}{\cos\theta}$$

$$+ (1 - r^2) \cos(\alpha_2 - \omega t)$$

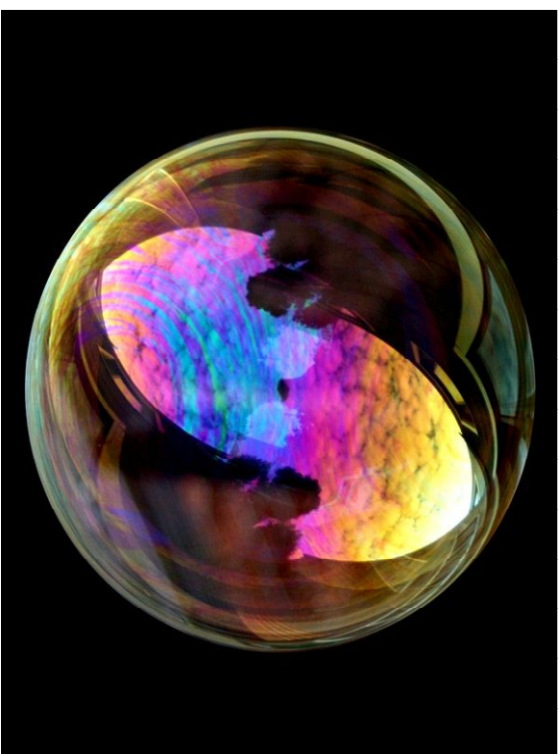
$$\frac{k_{nd}}{\cos\theta} = \frac{\pi}{2}, \frac{3\pi}{2}, \dots, \left(\frac{2m+1}{2} \right) \pi$$

$$d = \frac{\lambda_n}{4}, \frac{3\lambda_n}{4}, \dots$$

You have seen Fabry-Pérot interference before:



Oil slick

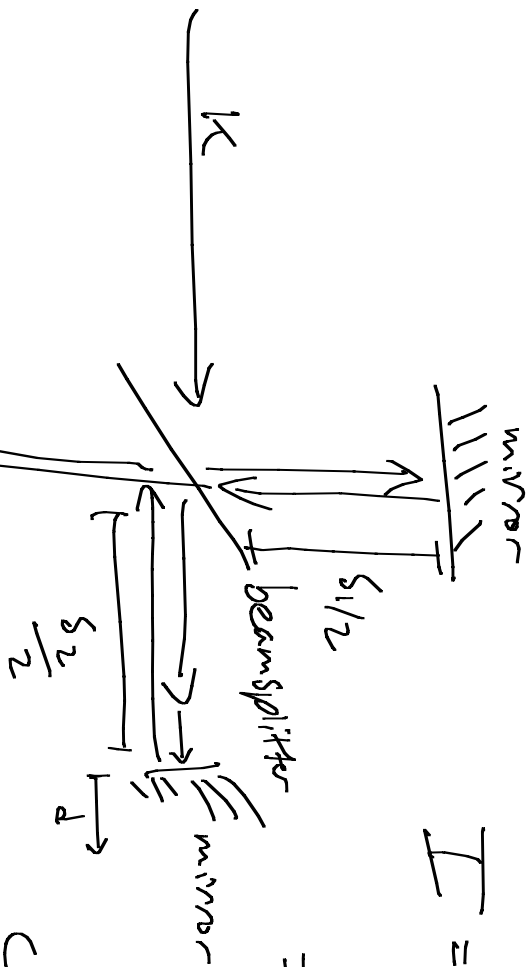


Soap bubble

Variations in thickness of the film changes the constructive / destructive conditions for different wavelengths.

However, in the pics above, you see a particular color when it is constructively reflected, not transmitted.

Example: Michelson Interferometer



$$I = \left(\frac{E_0}{2}\right)^2 + \left(\frac{E_0}{2}\right)^2 + 2 \frac{E_0}{2} \frac{E_0}{2} \cos(\alpha_1 - \alpha_2)$$

$$= \frac{E_0^2}{2} + \frac{E_0^2}{2} \cos(k(s_1 - s_2))$$

$$\text{IF } s_1 - s_2 = 2d$$

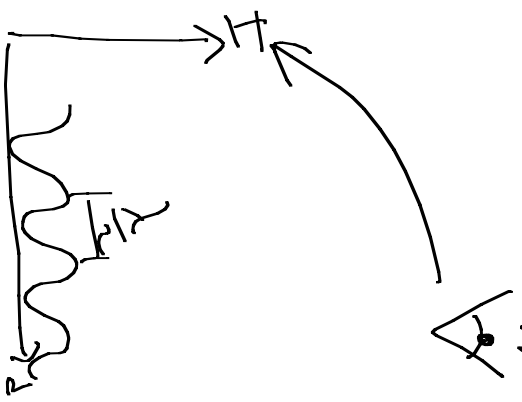
Constructive interference for

$$2kd = 0, 2\pi, \dots, 2m\pi \quad 2kd = 2m\pi$$

$$d = \frac{\pi}{k} = \frac{\lambda}{2}$$

destructive interference for

$$2kd = \pi, 3\pi, \dots, (2m+1)\pi$$



FFT

