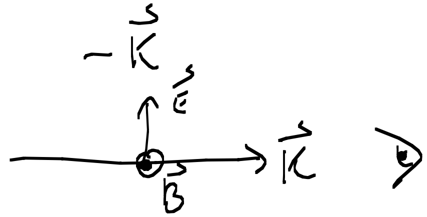
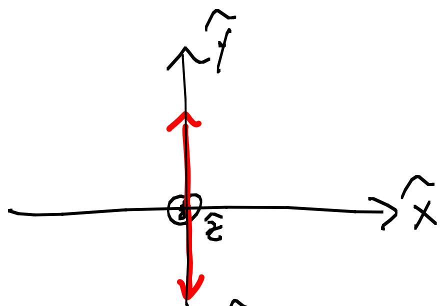
View Polarization

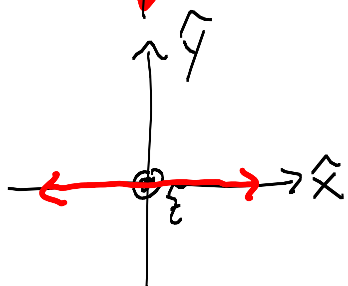
along $-\vec{k}$



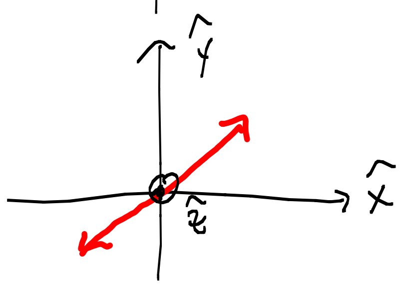
$\vec{k} = k\hat{z}$



$$\vec{E} = \mathrm{Re}\{E_{0y}\,\hat{y}\,e^{-i\omega t}\}$$



$$\vec{E} = \mathrm{Re}\{E_{0x}\,\hat{x}\,e^{-i\omega t}\}$$



$$\vec{E} = \mathrm{Re}\left\{\left(E_{0x}\hat{x} + E_{0y}\hat{y}\right)e^{-i\omega t}\right\}$$

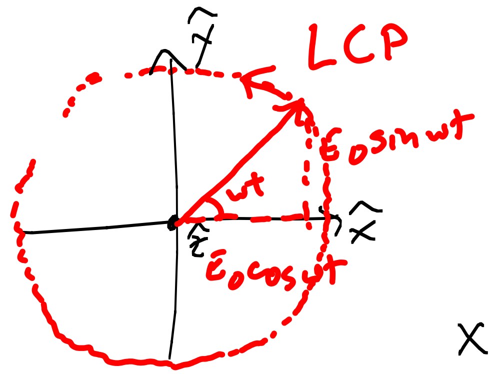
Superposition w/ Complex Coefficients

$E_{0x} = A \qquad E_{0y} = Be^{i\Delta\phi}$

$\Delta\phi$: phase diff. between E_{0x} and E_{0y}

$\Delta\phi = \phi_y - \phi_x \quad (-\pi < \Delta\phi < \pi)$

Example: $E_{0x} = E_0$, $E_{0y} = E_0 e^{i\Delta\phi}$, $\Delta\phi = \frac{\pi}{2}$ $\qquad e^{i\frac{\pi}{2}} = i$



$$\vec{E} = \mathrm{Re}\left\{E_0(\hat{x} + i\hat{y})e^{-i\omega t}\right\}$$

$$\mathrm{Re}\{A\} = \frac{A + A^*}{2}$$

$$x:\ \mathrm{Re}\left\{E_0 e^{-i\omega t}\right\} = E_0 \frac{e^{-i\omega t} + e^{i\omega t}}{2} = E_0 \cos\omega t$$

$$y:\ \mathrm{Re}\left\{E_0 i e^{-i\omega t}\right\} = E_0 \frac{ie^{-i\omega t} - ie^{+i\omega t}}{2} = E_0 \sin\omega t$$

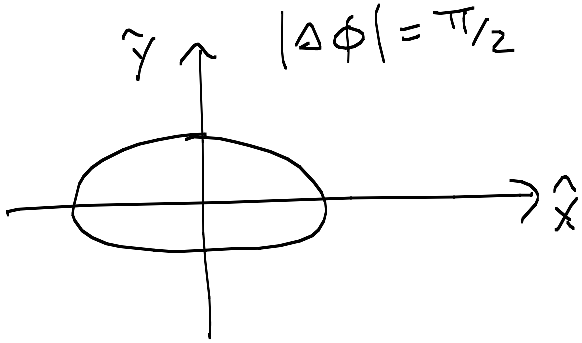
$$|\vec{E}| = \left(E_0^2\cos^2\omega t + E_0^2 \sin^2\omega t\right)^{1/2} = E_0$$

"circularly polarized"

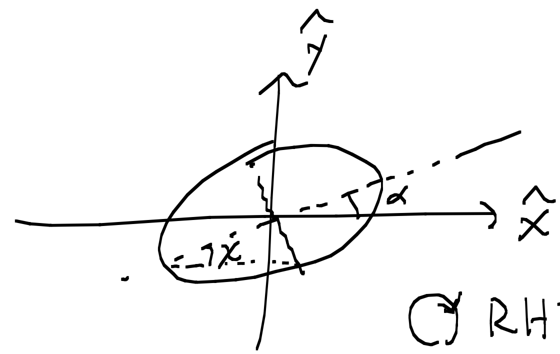
(RCP has $\Delta\phi = -\pi/2$)

General case:

$|E_{ox}| \neq |E_{oy}|$ elliptical polarization



$\xrightarrow{|\Delta\phi| \neq \pi/2}$



RHP $\Delta\phi < 0$

LHP $\Delta\phi > 0$

α: angle of inclination

Eqn 14-16: $\tan 2\alpha = \dfrac{2 E_{ox} E_{oy} \cos \Delta\phi}{E_{ox}^2 - E_{oy}^2}$

(deriv. added later)

χ: ellipticity

Nomenclature: Jones Vectors

$$\vec{E} = E_x \hat{x} + E_y \hat{y} \longrightarrow \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

e.g.

$$E_{0x} = |E_x| , \quad E_{0y} = |E_y| \longrightarrow \text{find } \alpha, \chi$$

$\hat{x}, \hat{y}$ basis is arbitrary!

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

Rotation matrix

example: use $+45°, -45°$ basis:

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Polarization Elements (polarizer, etc)

Matrices

$$\begin{bmatrix} A \\ B \end{bmatrix} \xrightarrow[\text{w/ TA} \parallel \hat{y}]{\text{polarizer}} \begin{bmatrix} 0 \\ B \end{bmatrix} \qquad \begin{bmatrix} A \\ B \end{bmatrix} \xrightarrow[\text{w/ TA} \parallel \hat{x}]{\text{Polarizer}} \begin{bmatrix} A \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ B \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ 0 \end{bmatrix}$$

Polarizer w/ TA at θ from vertical

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \sin^2\theta & \sin\theta \cdot \cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}$$

Phase Retarder

Birefringence: index of refraction is polarization dependent

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\Delta\phi} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ Be^{i\Delta\phi} \end{bmatrix}$$

Examples

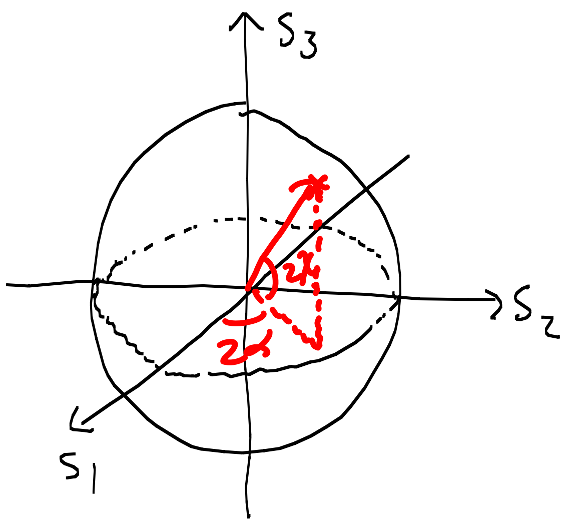
$\Delta\phi = \frac{\pi}{2}$ ("QWP") initially linear polarization @ 45°

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix} \longrightarrow$$ circular polarization!

$\Delta\phi = \pi$ ("HWP")

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \longrightarrow$$ linearly polarized @ $-45°$

Alternative nomenclature: Stokes parameters



$$S_0 = I \quad \text{(intensity)}$$

$$S_1 = I_p \cos 2\alpha \cos 2\chi = E_x^2 - E_y^2$$

$$S_2 = I_p \sin 2\alpha \cos 2\chi = E_{45}^2 - E_{-45}^2$$

$$S_3 = I_p \sin 2\chi = \underbrace{E_{LCP}^2 - E_{RCP}^2}$$

Birefringent elements transform one point to another on the surface of this sphere

QWP: rotations along polar direction (χ)

HWP: rotations along azimuthal direction (α)