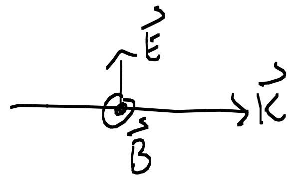
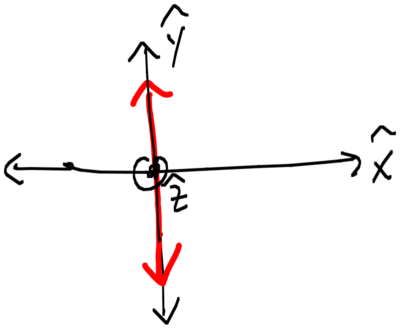
Polarization

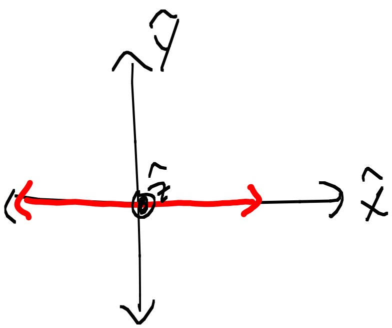


View along $-\vec{k}$

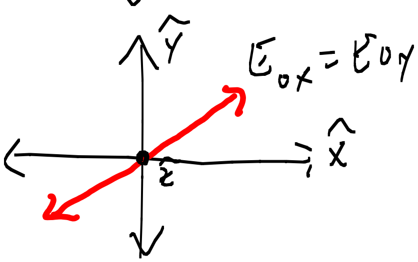
$\vec{k} \parallel \hat{z}$



$$\vec{E} = \mathrm{Re}\left\{E_{0y}\hat{y}\, e^{-i\omega t}\right\}$$



$$\vec{E} = \mathrm{Re}\left\{E_{0x}\hat{x}\, e^{-i\omega t}\right\}$$



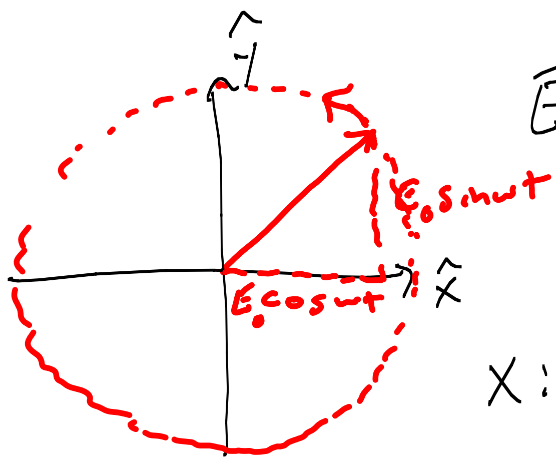
$$\vec{E} = \mathrm{Re}\left\{\left(E_{0x}\hat{x} + E_{0y}\hat{y}\right) e^{-i\omega t}\right\}$$

Superposition w/ Complex Coefs.

$$E_{0x} = A \qquad E_{0y} = B e^{i\Delta\phi}$$

$\Delta\phi$: phase difference betw. E_{0x} and E_{0y}

Example: $E_{0x} = E_0 \qquad E_{0y} = E_0 e^{i\Delta\phi}, \quad \Delta\phi = \frac{\pi}{2}$



LCP

$$\vec{E} = \mathrm{Re}\left\{\left(E_0\hat{x} + E_0 e^{i\frac{\pi}{2}}\hat{y}\right)e^{-i\omega t}\right\}$$

$$= \mathrm{Re}\left\{E_0(\hat{x} + i\hat{y})\, e^{-i\omega t}\right\}$$

$$x: \ \mathrm{Re}\left\{E_0 e^{-i\omega t}\right\} = E_0 \frac{e^{-i\omega t} + e^{i\omega t}}{2} = E_0 \cos\omega t$$

$$y: \ \mathrm{Re}\left\{E_0 i e^{-i\omega t}\right\} = E_0 \frac{ie^{-i\omega t} - i e^{i\omega t}}{2}$$

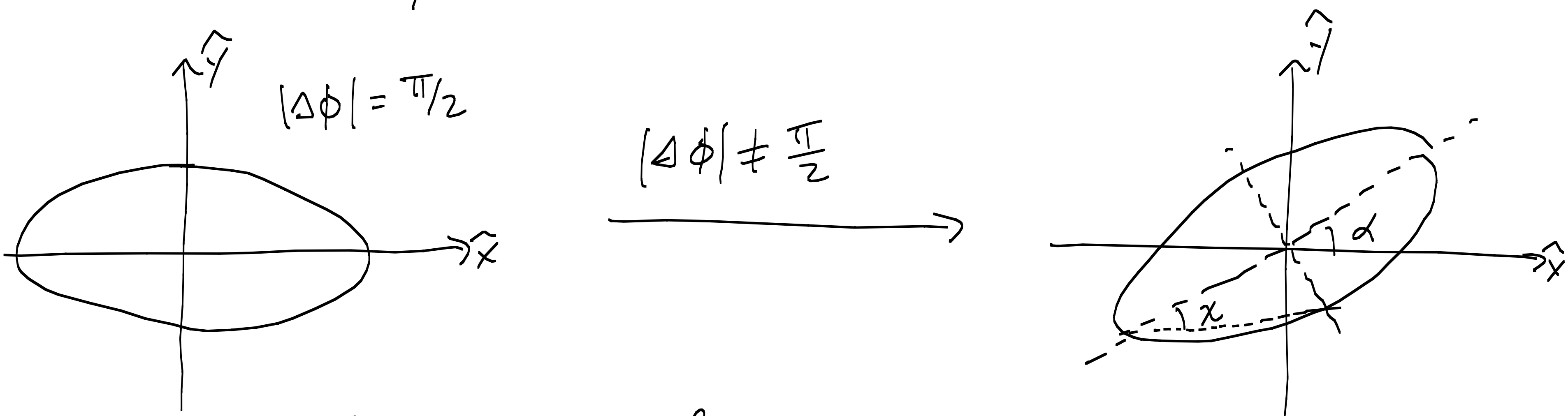
$$= E_0 \frac{e^{i\omega t} - e^{-i\omega t}}{2i} = E_0 \sin\omega t$$

(RCP by $\Delta\phi = -\frac{\pi}{2}$)

$$|\vec{E}| = \sqrt{E_0^2\cos^2\omega t + E_0^2 \sin^2\omega t} = E_0$$

General case:

$|E_{0x}| \neq |E_{0y}|$ $\quad \Delta\phi$ arbitrary



α: angle of inclination

χ: ellipticity

$\chi = 0$ linear

$\chi = \frac{\pi}{4}$ circular

Nomenclature: Jones vectors

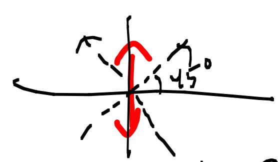
$$\vec{E} = \mathrm{Re}\{ E_x \hat{x} + E_y \hat{y} \} \longrightarrow \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

e.g. $E_{0x} = |E_x|$, $E_{0y} = |E_y|$ $\rightarrow$ find α, χ

Basis: $\hat{x}, \hat{y}$ basis $\begin{bmatrix} A \\ B \end{bmatrix}$

Rotation matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Example: use $+45°$, $-45°$



$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

← vertical linearly pol.

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Polarization Elements (polarizers, etc)

Matrices!

$$\begin{bmatrix} A \\ B \end{bmatrix} \xrightarrow[\text{TA} \parallel \hat{y}]{\text{polarizer}} \begin{bmatrix} 0 \\ B \end{bmatrix} \qquad \begin{bmatrix} A \\ B \end{bmatrix} \xrightarrow[\text{TA} \parallel \hat{x}]{\text{polarizer}} \begin{bmatrix} A \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ B \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ 0 \end{bmatrix}$$

Polarizer w/ TA at θ from vertical:

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}$$

Phase Retarder

Birefringence: n is polarization-dependent

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\Delta\phi} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ Be^{i\Delta\phi} \end{bmatrix}$$

Examples:

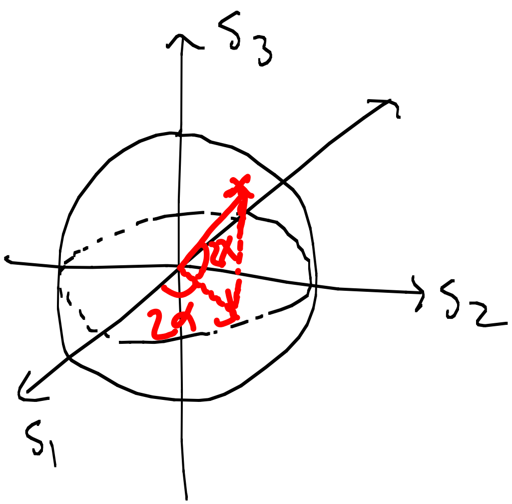
$\Delta\phi = \frac{\pi}{2}$ ("QWP") initial polarization linear @ 45°

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix} \longrightarrow \text{circular polarization}$$

$\Delta\phi = \pi$ ("HWP")

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \rightarrow \text{linearly polarized @ } -45^\circ$$

Alternative nomenclature: Stokes parameters



$$S_0 = I$$

$$S_1 = I_p \cos 2\alpha \cos 2\chi = E_x^2 - E_y^2$$

$$S_2 = I_p \sin 2\alpha \cos 2\chi = E_{45}^2 - E_{-45}^2$$

$$S_3 = I_p \sin 2\chi = \underbrace{E_{LCP}^2 - E_{RCP}^2}_{\text{experimentally measurable}}$$

QWP : rotation along polar direction (χ)

HWP : rotations along azimuthal direction (α)