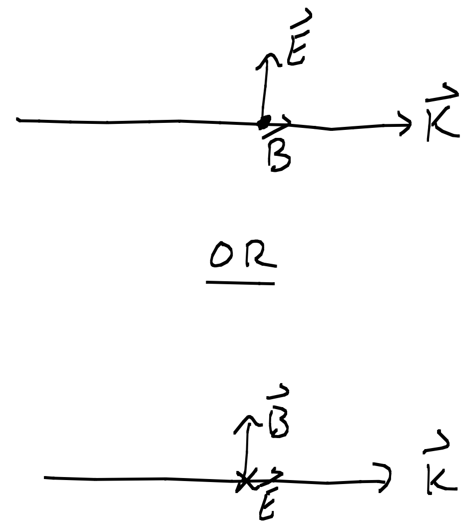
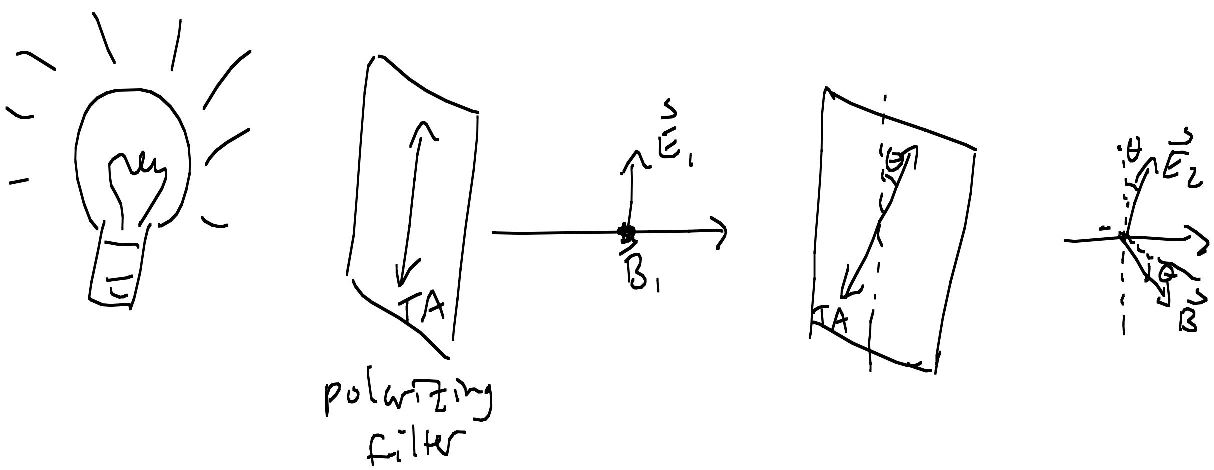
Polarization

plane wave:
transverse E, M fields
- 2 orthogonal polarization states



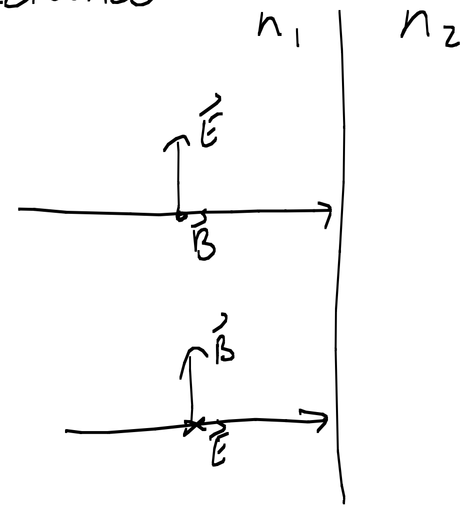


$$|\vec{E}_2| = |\vec{E}_1| \cos\theta$$

$$\frac{I_2}{I_1} = \cos^2\theta$$

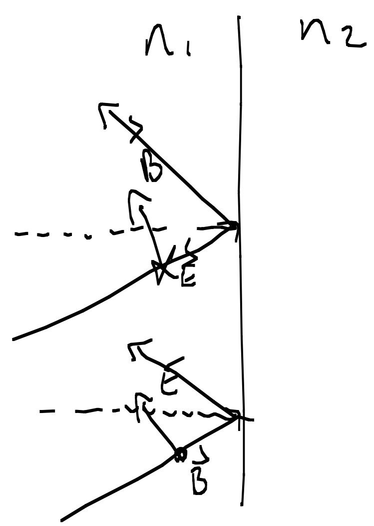
Interfaces

Normal Incidence:



configurations are equivalent!

Oblique:

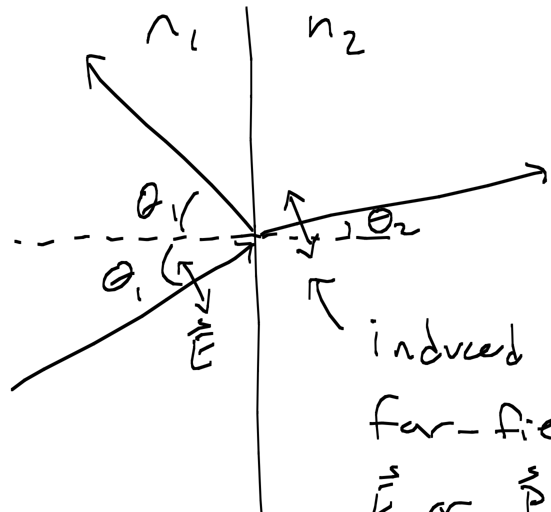


Transverse Electric (TE)

Transverse Magnetic (TM)

Oblique Incidence

$\boxed{\text{TM}}$

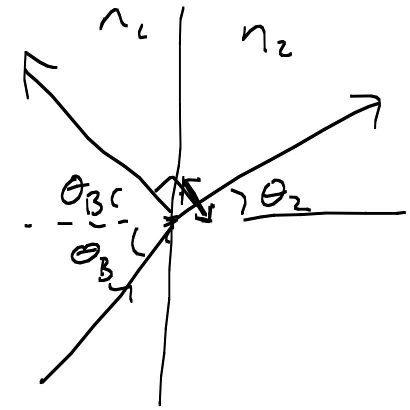


induced dipole far-field (radiated)

$$\vec{E} \propto \frac{\vec{p} \times \vec{r}}{r^2}$$

$$= 0 \quad \text{when} \quad \vec{r} \parallel \vec{p}$$

We expect zero reflection when



$$\theta_B + \theta_2 = \frac{\pi}{2}$$

$$\theta_2 = \frac{\pi}{2} - \theta_B$$

Snell's Law:

$$n_1 \sin\theta_B = n_2 \sin\theta_2$$

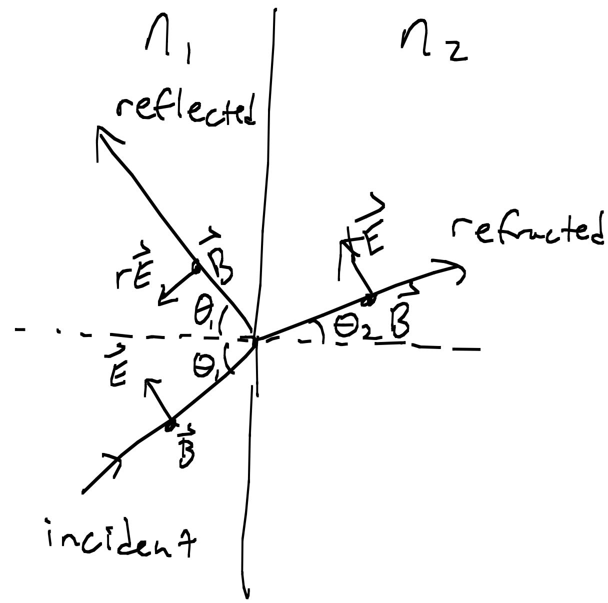
$$= n_2 \sin\left(\frac{\pi}{2} - \theta_B\right) = n_2 \cos\theta_B$$

$$\tan\theta_B = \frac{n_2}{n_1} \longrightarrow \theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$ Brewster's Angle

"Exact" treatment: Maxwell Eqn Boundary Conditions

Example:

TM



① $E_1^{\parallel} = E_2^{\parallel}$

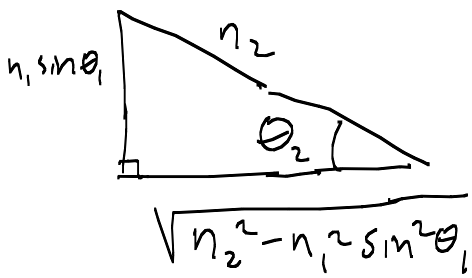
② $\dfrac{B_1^{\parallel}}{\mu_1} = \dfrac{B_2^{\parallel}}{\mu_2}$

$$\left[\, |B^{\parallel}| = \frac{n}{c}\, |E^{\parallel}| \,\right]$$

	$E^{\parallel}$	$B^{\parallel}$
incident:	$\lvert\vec{E}\rvert \cos\theta_1$	$\frac{n_1}{c}\lvert\vec{E}\rvert$
reflected:	$-\lvert\vec{E}\rvert\, r \cos\theta_1$	$\frac{n_1}{c}\, r\, \lvert\vec{E}\rvert$
refracted:	$t\lvert\vec{E}\rvert \cos\theta_2$	$\frac{n_2}{c}\, t\, \lvert\vec{E}\rvert$

① $E_1^{\parallel} = E_2^{\parallel}$:

$$|\vec{E}|\,(1-r)\cos\theta_1 = |\vec{E}|\,t\cos\theta_2$$



Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

$$\rightarrow \cos\theta_2 = \frac{\sqrt{n_2^2 - n_1^2\sin^2\theta_1}}{n_2}$$

$$(1-r)\cos\theta_1 = t\,\frac{\sqrt{n_2^2 - n_1^2\sin^2\theta_1}}{n_2}$$

② $\frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2}$ (Assume $\mu_1 = \mu_2 = 1$)

$$\frac{n_1}{c}|\vec{E}| + \frac{n_1}{c}r|\vec{E}| = \frac{n_2}{c}t|\vec{E}|$$

$$n_1(1+r) = n_2 t$$

$$r = \frac{n_2}{n_1}t - 1$$

Subst. ② into ①:

$$\left(2 - \frac{n_2}{n_1} t\right) \cos\theta_1 = t \, \frac{\sqrt{n_2^2 - n_1^2 \sin^2\theta_1}}{n_2}$$

$$2\cos\theta_1 = t\left(\frac{n_2}{n_1}\cos\theta_1 + \frac{\sqrt{n_2^2 - n_1^2 \sin^2\theta_1}}{n_2}\right)$$

$$t = \frac{2\cos\theta_1}{\frac{n_2^2}{n_1 n_2}\cos\theta_1 + \frac{n_1\sqrt{n_2^2 - n_1^2 \sin^2\theta_1}}{n_1 n_2}}$$

$$t_{TM} = \frac{2 n_2 \cos\theta_1}{\frac{n_2^2}{n_1}\cos\theta_1 + \sqrt{n_2^2 - n_1^2 \sin^2\theta_1}}$$

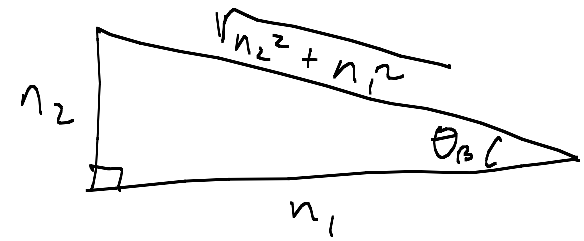
$$r_{TM} = 1 - \frac{n_2}{n_1} t_{TM}$$

c.f. Eqn 23-30
23-28

"Fresnel Eqns"

What happens when $\theta_1 = \theta_B$?

$$\theta_B = \tan^{-1} \frac{n_2}{n_1}$$



$$\cos\theta_B = \frac{n_1}{\sqrt{\ }} \quad , \quad \sin\theta_B = \frac{n_2}{\sqrt{\ }}$$

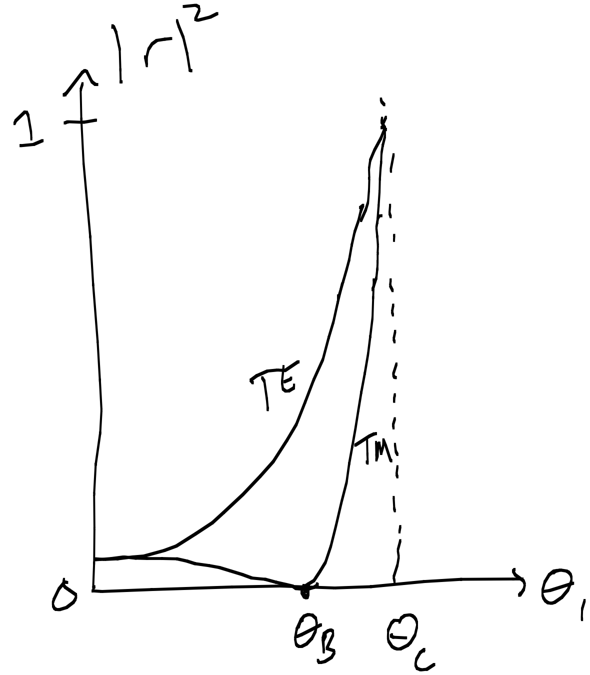
$$t_{TM}(\theta_1 = \theta_B) = \frac{2 n_2 \frac{n_1}{\sqrt{\ }}}{\frac{n_2^2}{n_1} \frac{n_1}{\sqrt{\ }} + \sqrt{n_2^2 - n_1^2 \frac{n_2^2}{n_1^2 + n_2^2}}}$$

$$= \frac{\frac{2 n_1 n_2}{\sqrt{\ }}}{\frac{n_2^2}{\sqrt{\ }} + \sqrt{\frac{\cancel{n_2^2 n_1^2} + n_2^4 - \cancel{n_1^2 n_2^2}}{n_1^2 + n_2^2}}} = \frac{n_1}{n_2}$$

$$r_{TM}(\theta_1 = \theta_B) = 1 - \frac{n_2}{n_1}\left(\frac{n_1}{n_2}\right) = 0 \ !$$

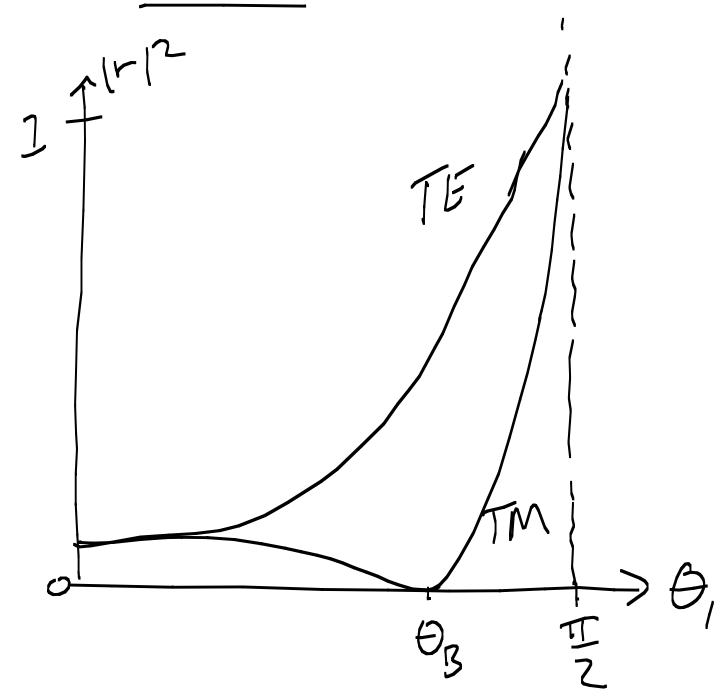
as expected!

$n_1 > n_2$



$|r|^2$
1
0
TE
TM
θ_B
θ_c
θ_1

$n_2 > n_1$



$|r|^2$
1
0
TE
TM
θ_B
$\frac{\pi}{2}$
θ_1