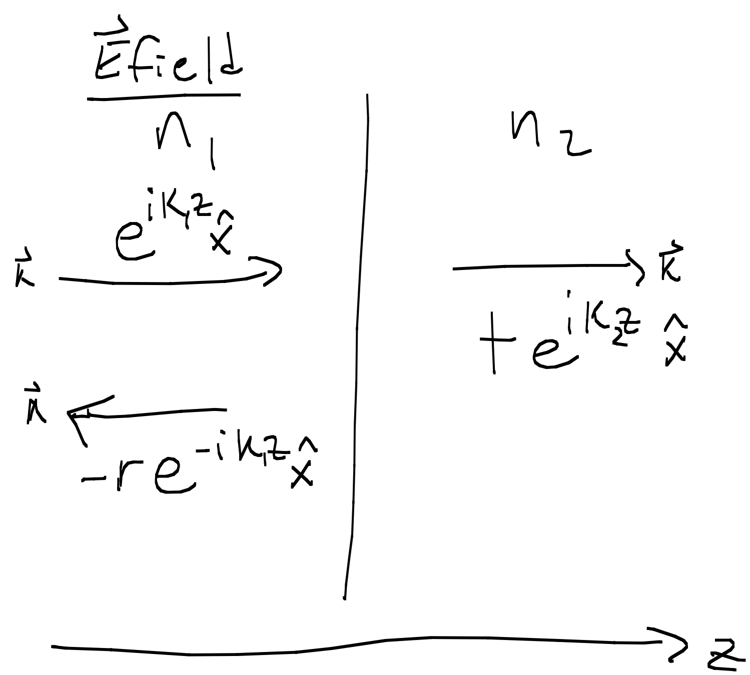
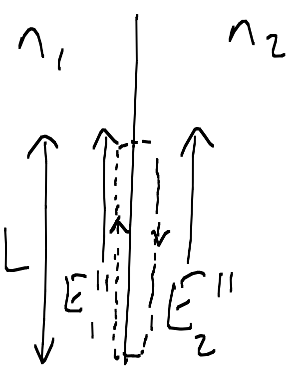
Reflection





$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$E_1^{\parallel}L - E_2^{\parallel}L = 0$$

$$E_1^{\parallel} = E_2^{\parallel}$$

We need another B.C.

→ From Maxwell Eqn's!:

$$\vec{\nabla}\cdot\vec{E} = 0$$
$$\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$$
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{B} = +\mu\varepsilon\frac{d\vec{E}}{dt}$$

Plane Wave $\vec{E}, \vec{B}$

$$\vec{E} = E_0 e^{i(kz-\omega t)} \hat{x}$$

Faraday's Law $\quad \vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ E_0 e^{i(kz-\omega t)} & 0 & 0 \end{vmatrix} = 0\hat{x} - \left(0 - ikE_0 e^{i(kz-\omega t)}\right)\hat{y} + 0\hat{z}$$

$$= ikE_0 e^{i(kz-\omega t)} \hat{y}$$

$$\vec{B} = \left(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}\right) e^{i(kz-\omega t)}$$

$$-\frac{d\vec{B}}{dt} = -\left[-i\omega\left(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}\right)e^{i(kz-\omega t)}\right] = ikE_0 e^{i(kz-\omega t)}\hat{y}$$

$$B_x = B_z = 0$$

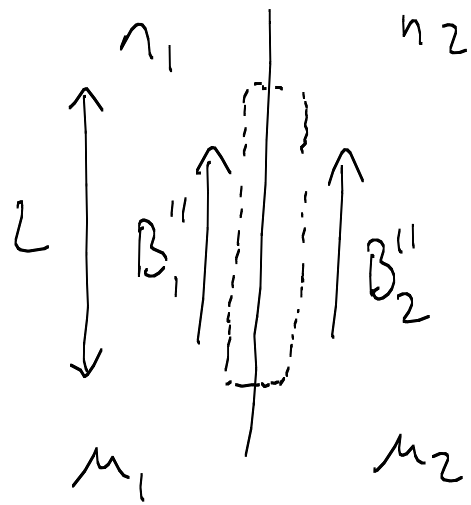
$$i\omega B_y e^{i(kz-\omega t)}\hat{y} = ikE_0 e^{i(kz-\omega t)}\hat{y}$$

$$B_y = \frac{k}{\omega}E_0 = \frac{E_0}{v} = \frac{E_0}{c/n} = \frac{nE_0}{c}$$

So, plane wave is TEM ($E \perp B$), and

$\vec{B}$ is in-phase w/ $\vec{E}$

Boundary Condition for B



$$\oint \vec{B} \cdot \vec{dl} = \mu I + \cancel{\mu \varepsilon \frac{d\Phi_E}{dt}} \rightarrow 0 \quad \text{(no Area)}$$

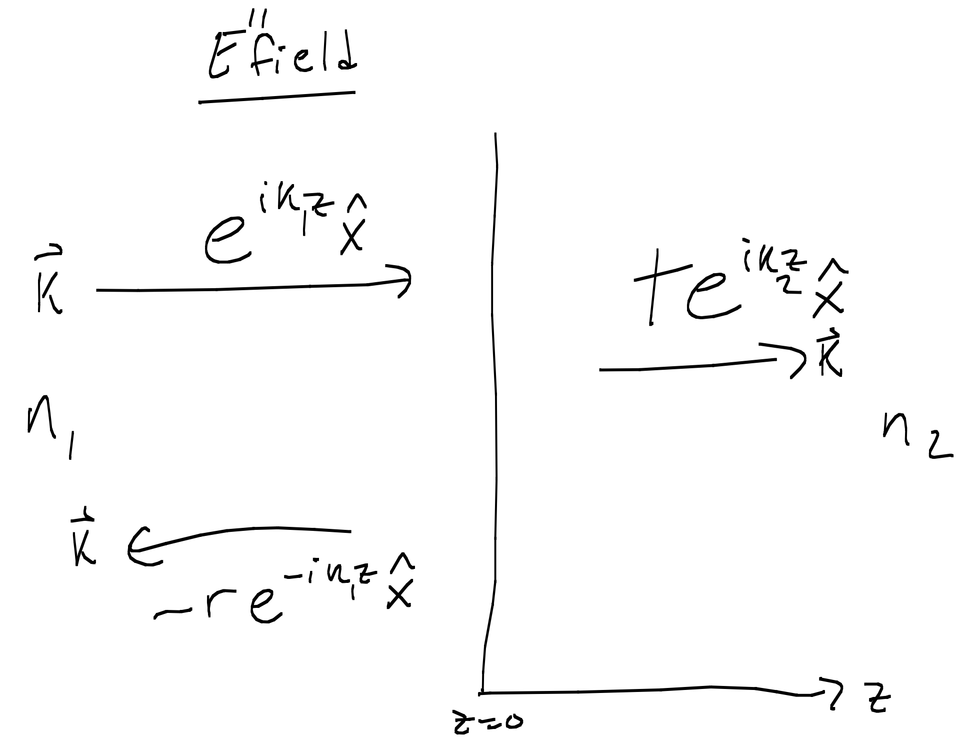
$$\oint \frac{\vec{B}}{\mu} \cdot \vec{dl} = 0$$

$$\frac{B_1^{\parallel}}{\mu_1} L - \frac{B_2^{\parallel}}{\mu_2} L = 0$$

$$\frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2}$$

$$\left(H_1^{\parallel} = H_2^{\parallel} \right)$$

① $E_1^{\parallel} = E_2^{\parallel}$ ② $\frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2}$ $\left(B^{\parallel} = \frac{n}{c} E^{\parallel} \right)$



① $e^{ikz} - re^{-ikz}\Big|_{z=0} = te^{ikz}\Big|_{z=0}$

② $\frac{n_1}{c}\frac{e^{ikz}}{\mu_1} + r\frac{n_1}{c}\frac{e^{-ikz}}{\mu_1}\Big|_{z=0} = t\,\frac{n_2}{c}\frac{e^{ikz}}{\mu_2}\Big|_{z=0}$

1 $\quad 1 - r = t$

2 $\quad \dfrac{n_1}{\mu_1} + r\dfrac{n_1}{\mu_1} = t\dfrac{n_2}{\mu_2}$

$$\frac{n_1}{\mu_1} + r\frac{n_1}{\mu_1} = (1-r)\frac{n_2}{\mu_2}$$

$$\frac{n_1}{\mu_1} = \frac{n_2}{\mu_2} - r\left(\frac{n_2}{\mu_2} + \frac{n_1}{\mu_1}\right)$$

$$r = -\frac{\frac{n_1}{\mu_1} - \frac{n_2}{\mu_2}}{\frac{n_1}{\mu_1} + \frac{n_2}{\mu_2}}$$

$$\frac{\mu c}{n} = \frac{\mu}{\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}$$ "intrinsic impedance"

for $\mu_1 = \mu_2$

$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

for $I = |E|^2$

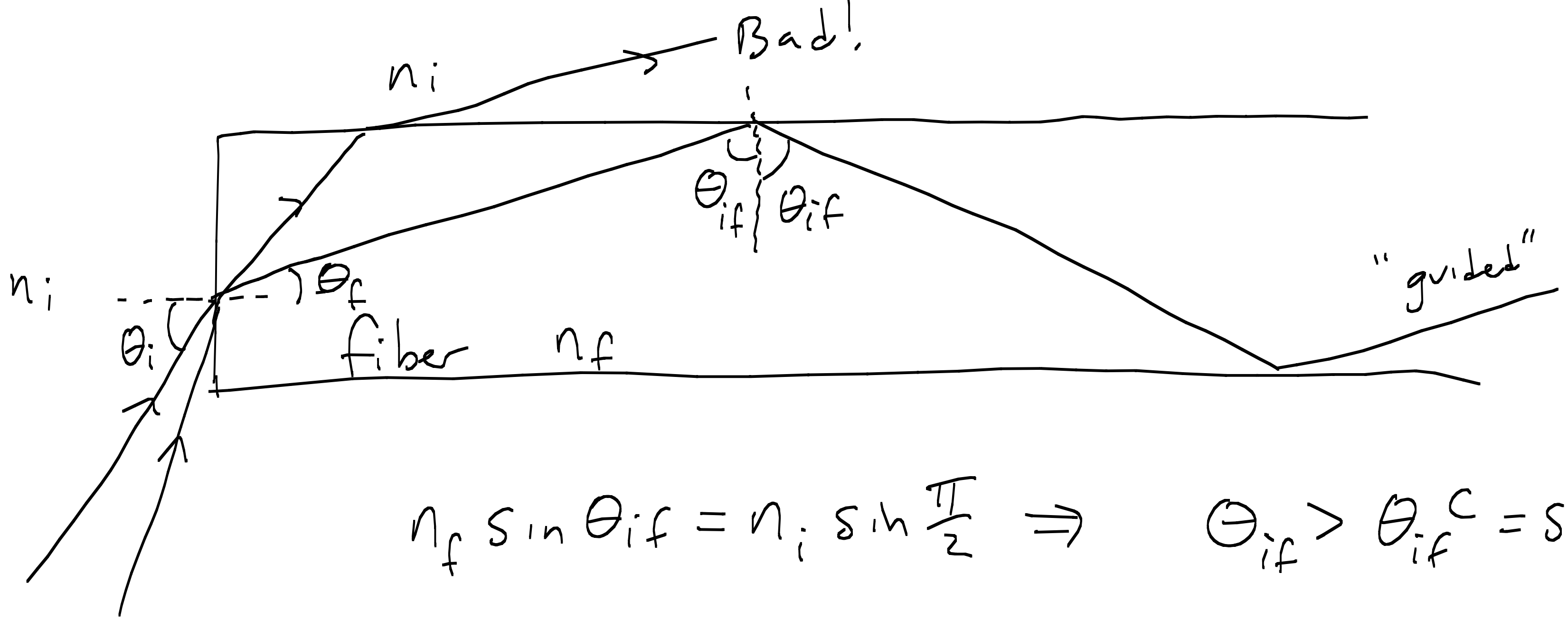
$$R = |r|^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = 1 - R = 1 - \frac{n_1^2 + n_2^2 - 2n_1n_2}{(n_1 + n_2)^2}$$

$$= \frac{\cancel{n_1^2} + \cancel{n_2^2} + 2n_1n_2 - (\cancel{n_1^2} + \cancel{n_2^2} - 2n_1n_2)}{(n_1+n_2)^2}$$

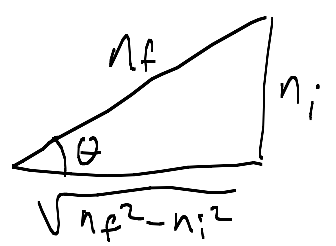
$$= \frac{4n_1n_2}{(n_1+n_2)^2}$$

TIR application: Fiber optics



$$n_f \sin\theta_{if} = n_i \sin\frac{\pi}{2} \Rightarrow \quad \theta_{if} > \theta_{if}^c = \sin^{-1}\frac{n_i}{n_f}$$

$$\sin\theta_f = \cos\theta_{if}$$

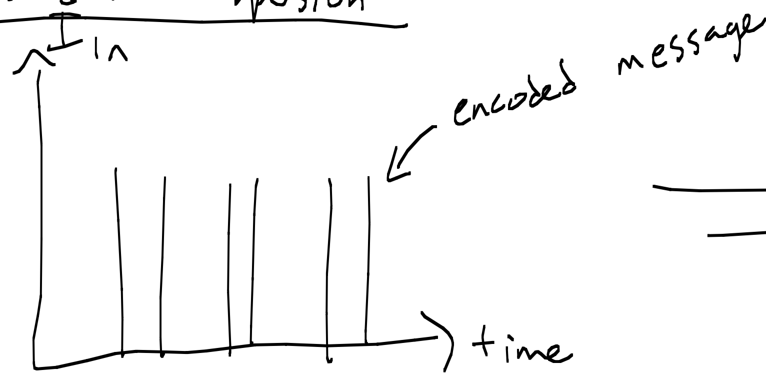


$$n_i \sin\theta_i^c = n_f \sin\theta_f = n_f \cos\theta_{if} = n_f \cos\left[\sin^{-1}\frac{n_i}{n_f}\right]$$

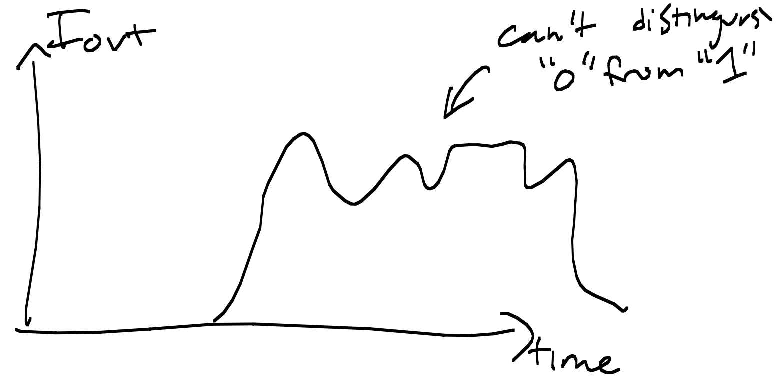
$$n_i \sin\theta_i^c = \cancel{n_f}\,\frac{\sqrt{n_f^2 - n_i^2}}{\cancel{n_f}}$$

for "guided" ray, $\theta_i < \theta_i^c = \sin^{-1}\left[\frac{\sqrt{n_f^2 - n_i^2}}{n_i}\right] = \sin^{-1}\left[\sqrt{\frac{n_f^2}{n_i^2} - 1}\right]$

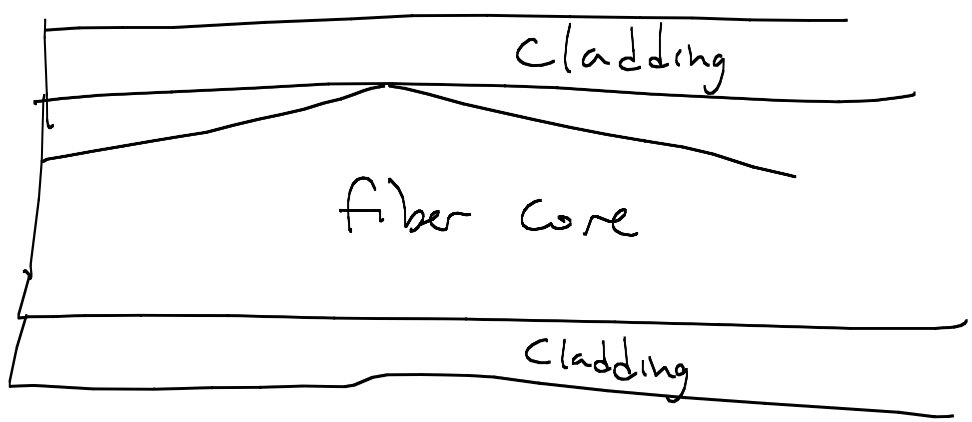
Intermodal dispersion



$\Longrightarrow$

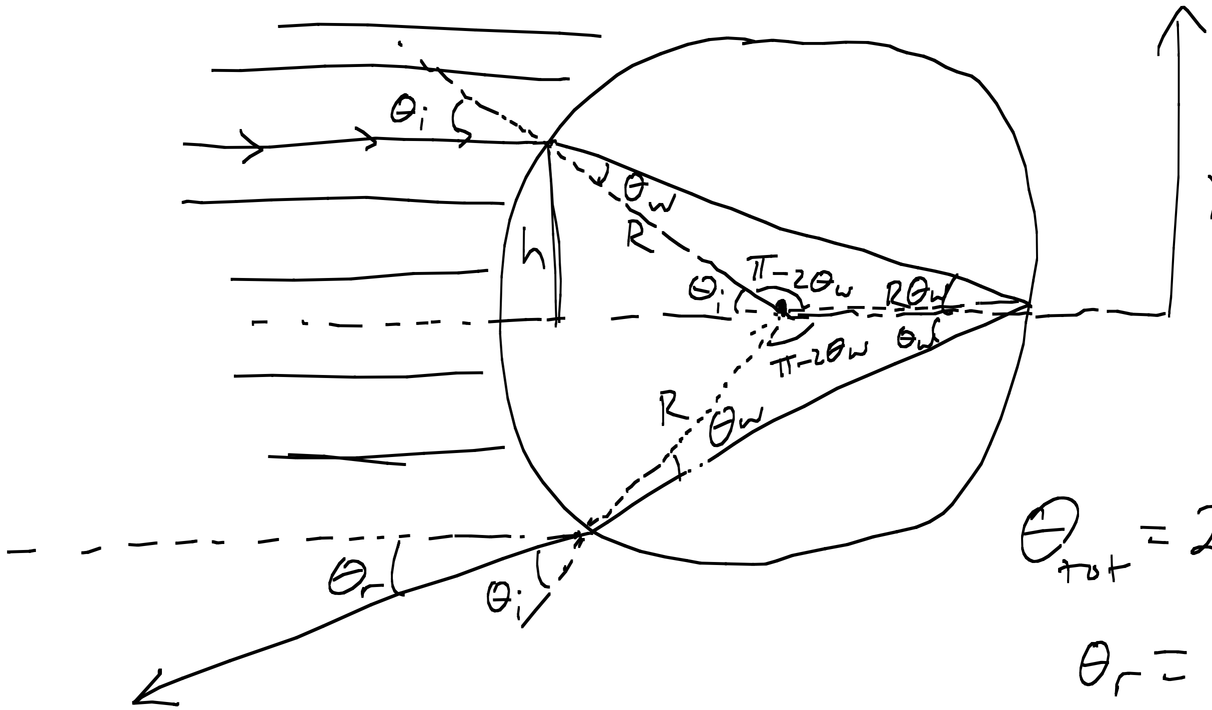


Solution



$n_{cladding} < \sim n_{core}$

Refraction + Reflection in a sphere



$$\theta_{tot} = 2(\pi - 2\theta_w) + 2\theta_i$$

$$\theta_r = 2\pi - (2\pi - 4\theta_w + 2\theta_i)$$

$$= 4\theta_w - 2\theta_i$$

$$= 4\sin^{-1}\left(\frac{\sin\theta_i}{n_w}\right) - 2\theta_i$$

$$0 < \theta_i < \frac{\pi}{2}$$

Illumination is const. as a function of $0 < h < R$

Caustics

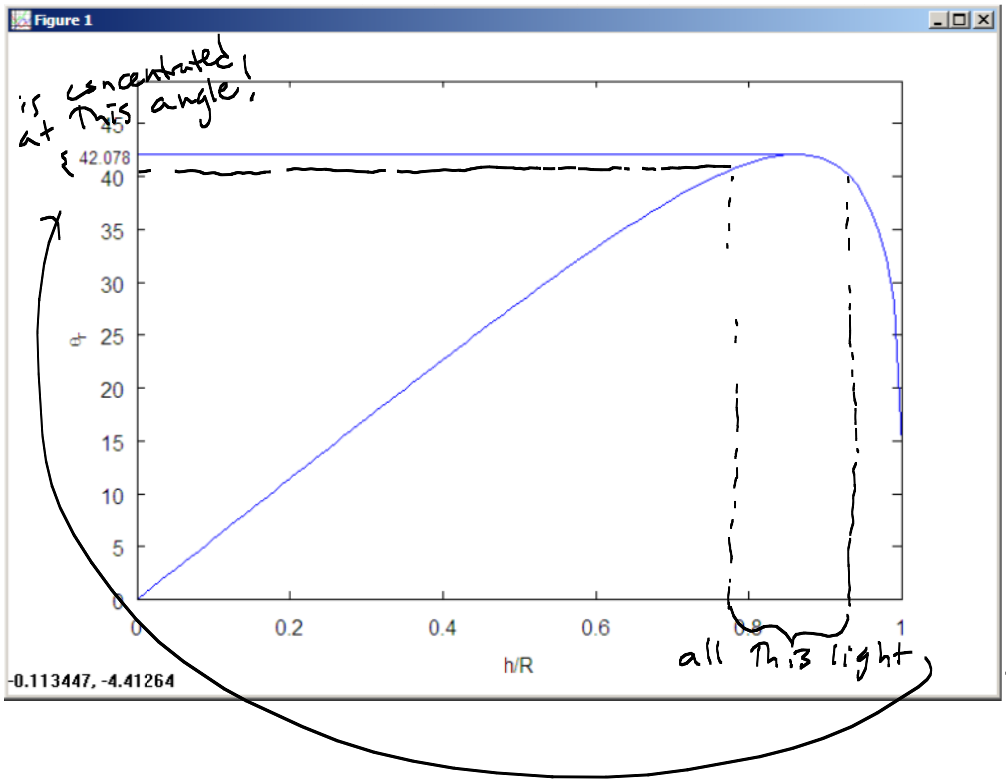


Figure 1
is concentrated at this angle!
42.078
all this light
h/R
-0.113447, -4.41264

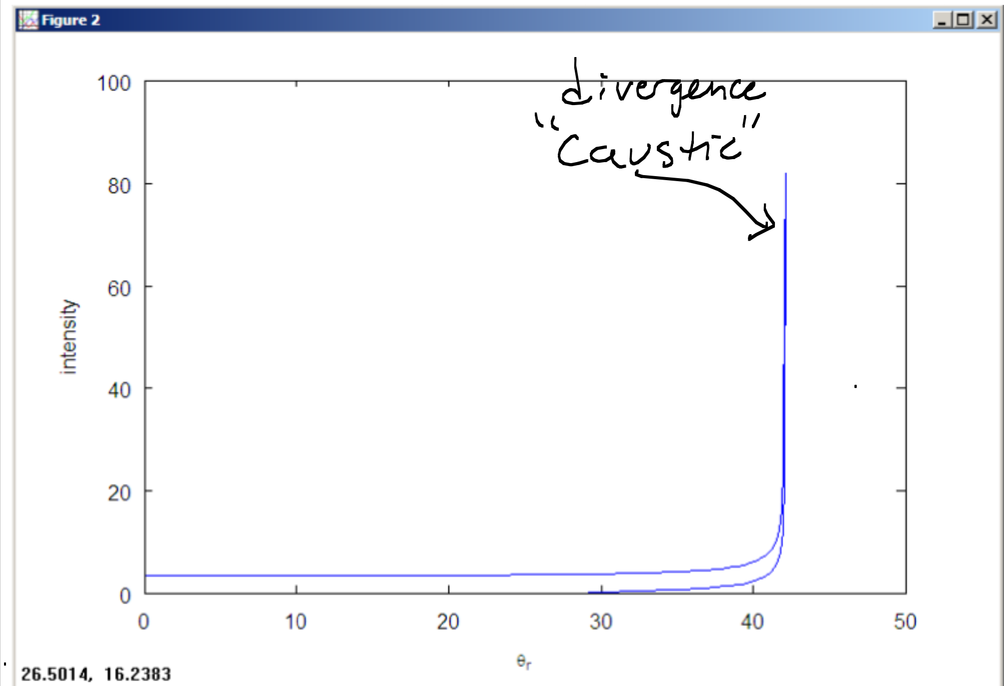
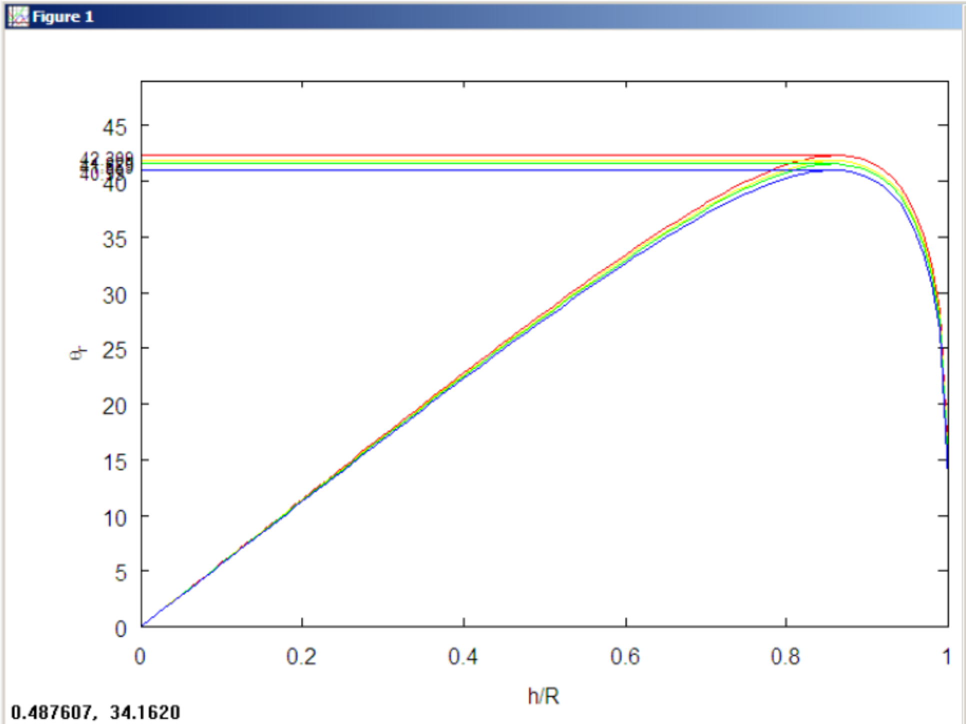
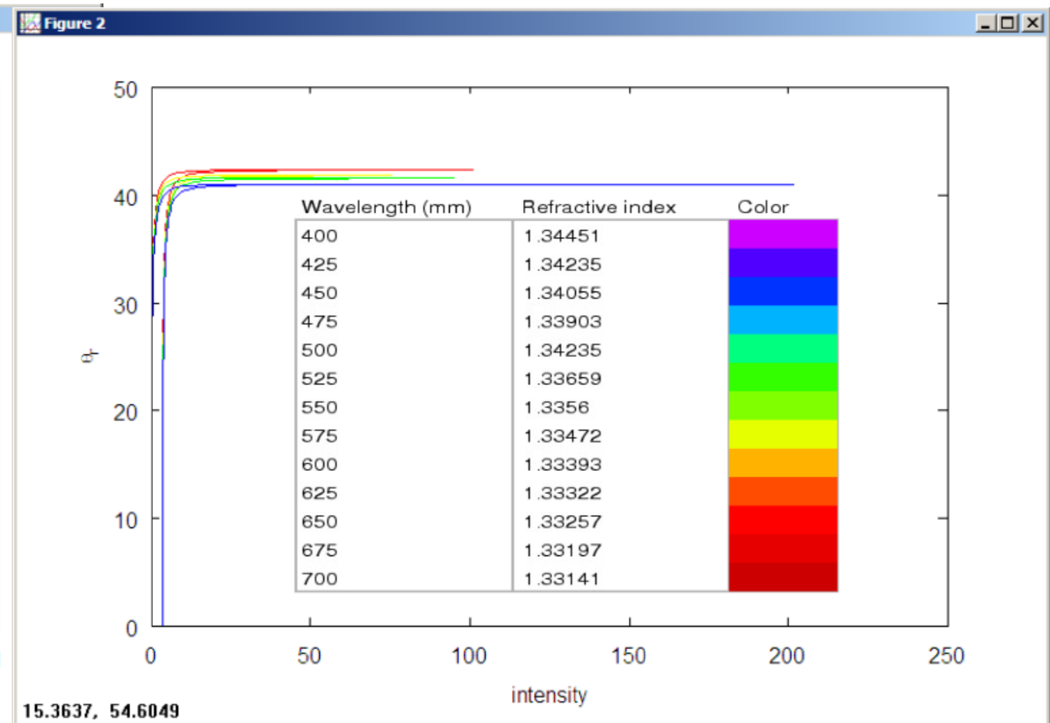


Figure 2
divergence
"Caustic"
intensity
26.5014, 16.2383

Rainbows





Second rainbow from 2 internal reflections → colors reversed!

primary rainbow