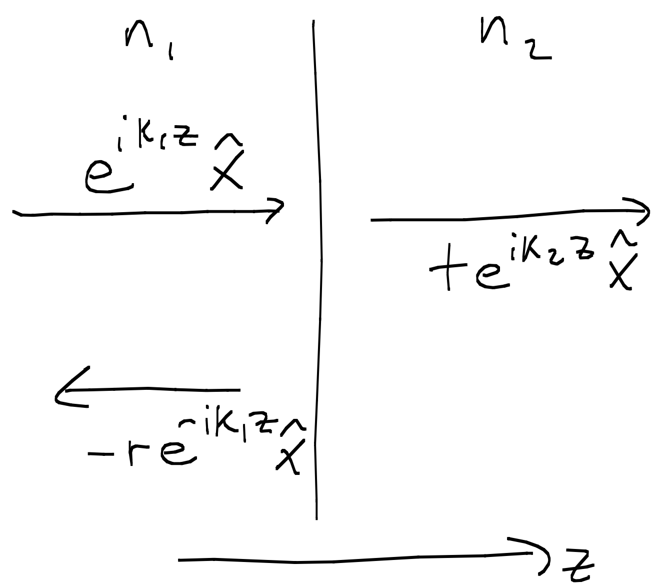
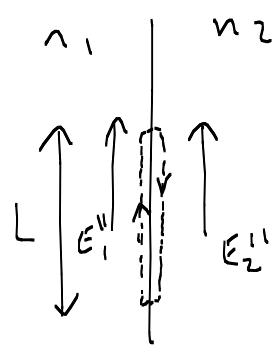
Reflection

$\vec{E}$ - field



Boundary condition:



$$\oint \vec{E}\cdot d\vec{l} = -\cancel{\frac{d\Phi_B}{dt}} \to 0$$

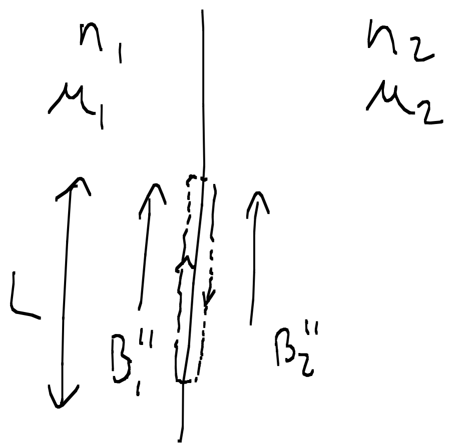
$$E_1^{\parallel}L - E_2^{\parallel}L = 0$$

$$E_1^{\parallel} = E_2^{\parallel}$$

we have 2 unknowns,

→ we need 2 B.C.'s!

B.C. for Magnetic field



$$\oint \vec{B} \cdot d\vec{l} = \mu I + \mu \varepsilon \cancel{\frac{d \Phi_E}{dt}} \rightarrow 0$$

$$\oint \frac{\vec{B}}{\mu} \cdot d\vec{l} = 0$$

$$\frac{B_1^{\parallel}}{\mu_1} L - \frac{B_2^{\parallel}}{\mu_2} L = 0$$

$$\frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2}$$

$$\left(H_1^{\parallel} = H_2^{\parallel} \right)$$

Plane wave $\vec{B}$ Components

$$\vec{E} = E_0 e^{i(kz-wt)}\hat{x}$$

Faraday's Law: $\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ E_0 e^{i(kz-wt)} & 0 & 0 \end{vmatrix} = 0\hat{x} - \left(0 - ikE_0 e^{i(kz-wt)}\right)\hat{y} + 0\hat{z}$$

$$= ikE_0 e^{i(kz-wt)}\hat{y}$$

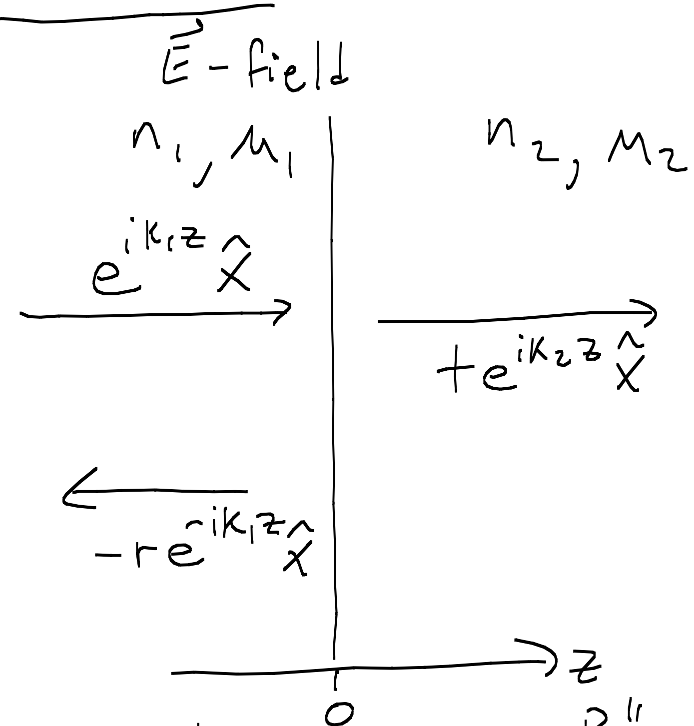
$$\vec{B} = \left(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}\right)e^{i(kz-wt)}$$

$$-\frac{d\vec{B}}{dt} = -\left(-iw\left(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}\right)e^{i(kz-wt)}\right)$$

$\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$: $\quad ikE_0 e^{i(kz-wt)}\hat{y} = iwB_y e^{i(kz-wt)}\hat{y}$

$$B_y = \frac{k}{w}E_0 = \frac{E_0}{v} = \frac{nE_0}{c}$$

Imposing Boundary Conditions



B.C.'s: ① $E_1^{\parallel} = E_2^{\parallel}$ ② $\frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2}$ $\left(B^{\parallel} = \frac{n}{c}E^{\parallel}\right)$

① : $e^{ik_1 z} - r e^{-ik_1 z}\Big|_{z=0} = t e^{ik_2 z}\Big|_{z=0} \longrightarrow 1 - r = t$

② : $\frac{n_1}{\mu_1 c} e^{ik_1 z} - \frac{n_1}{\mu_1 c} r e^{-ik_1 z}\Big|_{z=0} = \frac{n_2}{\mu_2 c} t e^{ik_2 z}\Big|_{z=0} \longrightarrow \frac{n_1}{\mu_1} - r\frac{n_1}{\mu_1} = \frac{n_2}{\mu_2}t$

$$\frac{n_1}{\mu_1} - r\frac{n_1}{\mu_1} = \frac{n_2}{\mu_2}(1-r)$$

$$\frac{n_1}{\mu_1} = \frac{n_2}{\mu_2} - r\left(\frac{n_1}{\mu_1} + \frac{n_2}{\mu_2}\right)$$

$$r = \frac{\frac{n_2}{\mu_2} - \frac{n_1}{\mu_1}}{\frac{n_1}{\mu_1} + \frac{n_2}{\mu_2}}$$

We measure Intensity $= |E|^2$, not E-field!

$$R = |r|^2 = \left(\frac{\frac{n_2}{\mu_2} - \frac{n_1}{\mu_1}}{\frac{n_1}{\mu_1} + \frac{n_2}{\mu_2}}\right)^2$$

$$\boxed{\frac{\mu c}{n} = \frac{\mu}{\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}}$$

"Intrinsic Impedance"

When $\mu_1 = \mu_2$,

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$R + T = 1 \qquad T = 1 - R$$

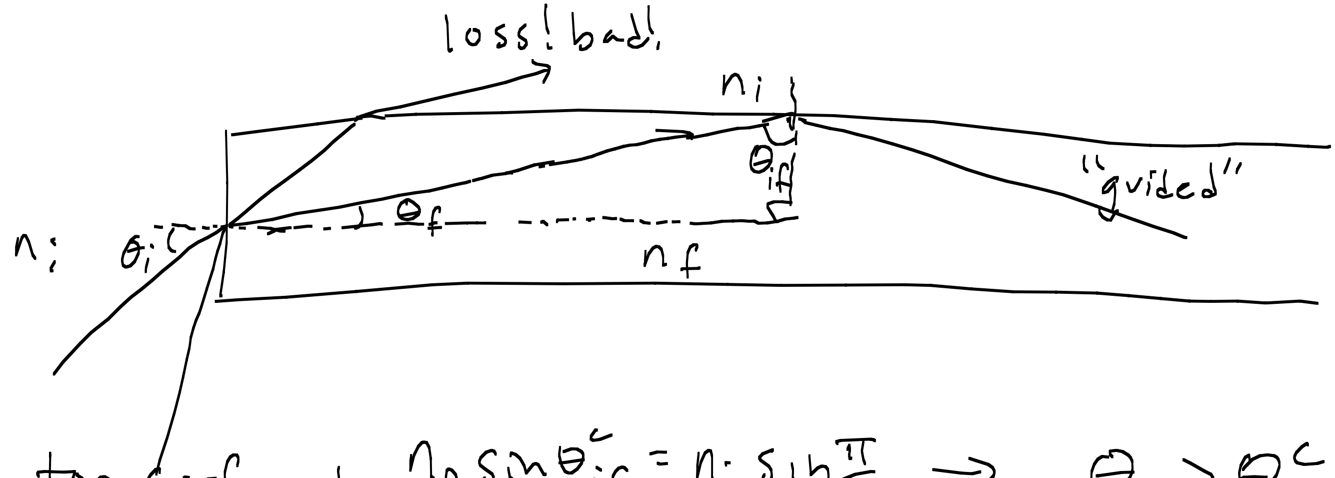
$$T = 1 - \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \frac{(n_1 + n_2)^2}{(n_1 + n_2)^2} - \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

$$= \frac{n_1^2 + n_2^2 + 2n_1 n_2 - (n_1^2 + n_2^2 - 2n_1 n_2)}{(n_1 + n_2)^2}$$

$$= \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

Reflection is consequence of field conservation at the interface!

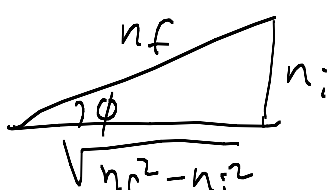
TIR application: fiber optics



top surface: (sides of fiber) $n_f \sin\theta_{if}^c = n_i \sin\frac{\pi}{2} \rightarrow \theta_{if} > \theta_{if}^c = \sin^{-1}\frac{n_i}{n_f}$

$$\sin\theta_f = \cos\theta_{if}$$

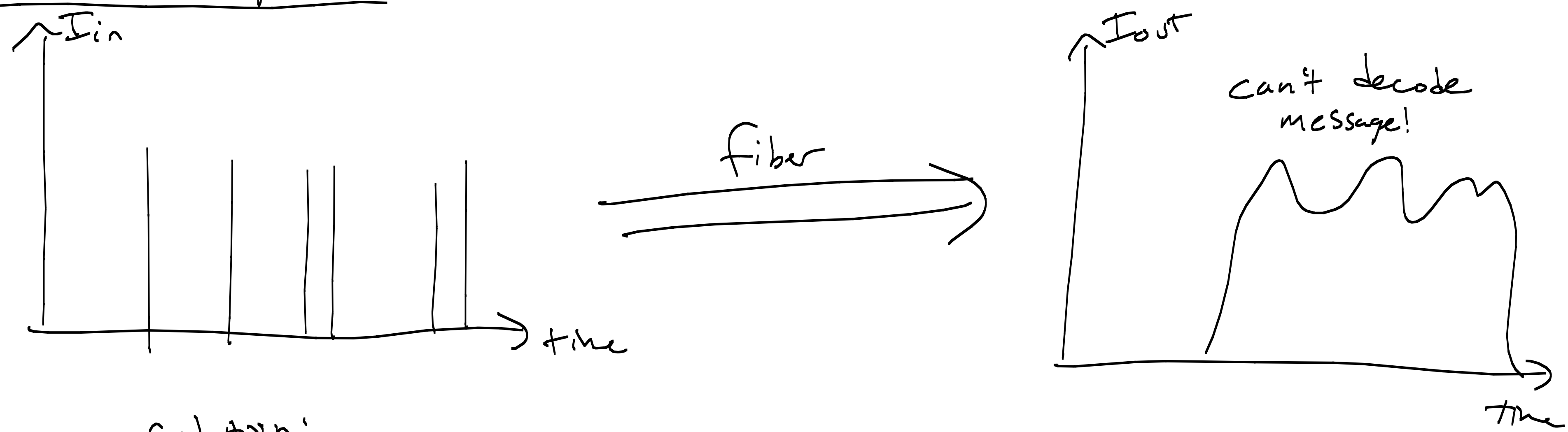
Input interface: $n_i \sin\theta_i^c = n_f \sin\theta_f^c = n_f \cos\theta_{if}^c = n_f \cos\left(\sin^{-1}\frac{n_i}{n_f}\right)$



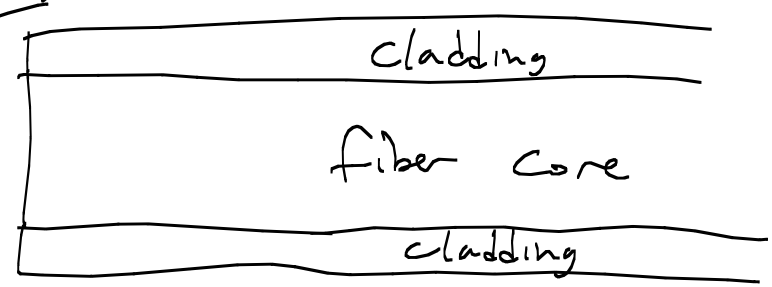
$$n_i \sin\theta_i^c = n_f \frac{\sqrt{n_f^2-n_i^2}}{n_f}$$

$$\theta_i < \theta_i^c = \sin^{-1}\frac{\sqrt{n_f^2-n_i^2}}{n_i} = \sin^{-1}\sqrt{\left(\frac{n_f}{n_i}\right)^2-1}$$

Intermodal dispersion



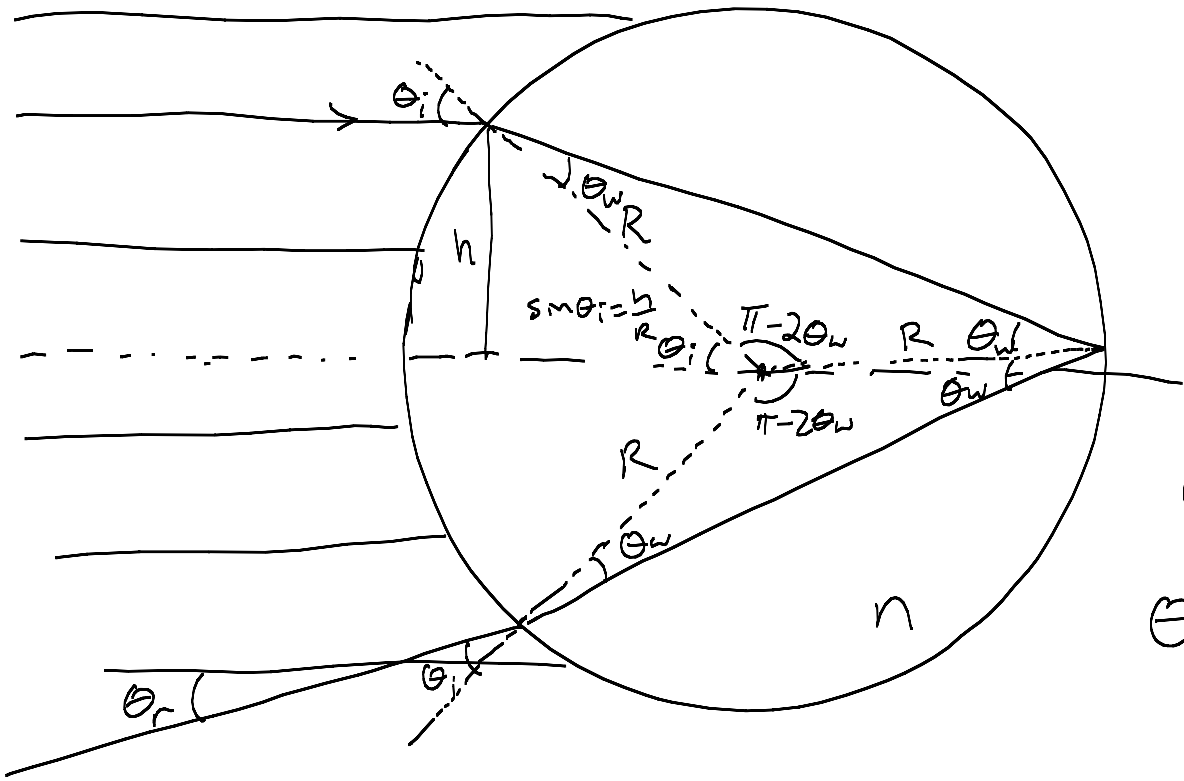
Solution:



$n_{cladding} \sim< n_{core}$

reduces θ_i^c and eliminates "intermodal dispersion"

Refraction and Reflection in a sphere



$$\theta_{tot} = 2(\pi - 2\theta_w) + 2\theta_i$$

$$\theta_r = 2\pi - \theta_{tot} = 4\theta_w - 2\theta_i$$

$$= 4\sin^{-1}\left(\frac{\sin\theta_i}{n}\right) - 2\theta_i$$

$$\theta_i = \sin^{-1}\frac{h}{R}$$

$$\sin\theta_i = n\sin\theta_w$$

$$\theta_w = \sin^{-1}\frac{\sin\theta_i}{n}$$

Caustics

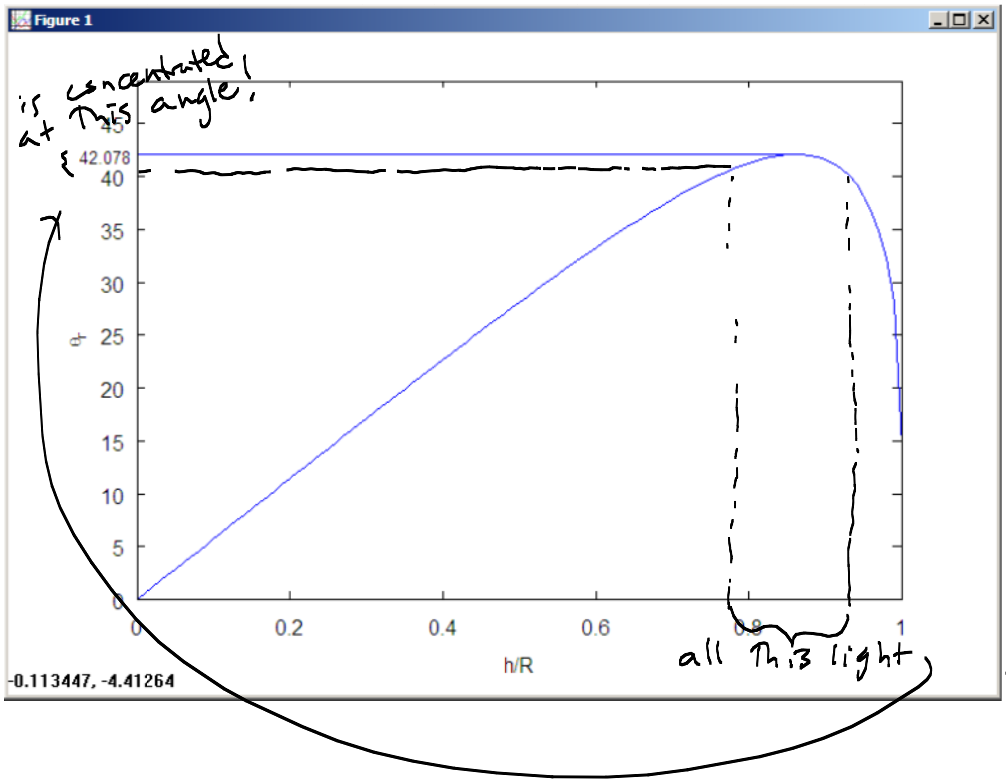


Figure 1
is concentrated at this angle!
42.078
45
40
35
30
25
20
15
10
5
0
0.2
0.4
0.6
0.8
1
h/R
all this light
-0.113447, -4.41264

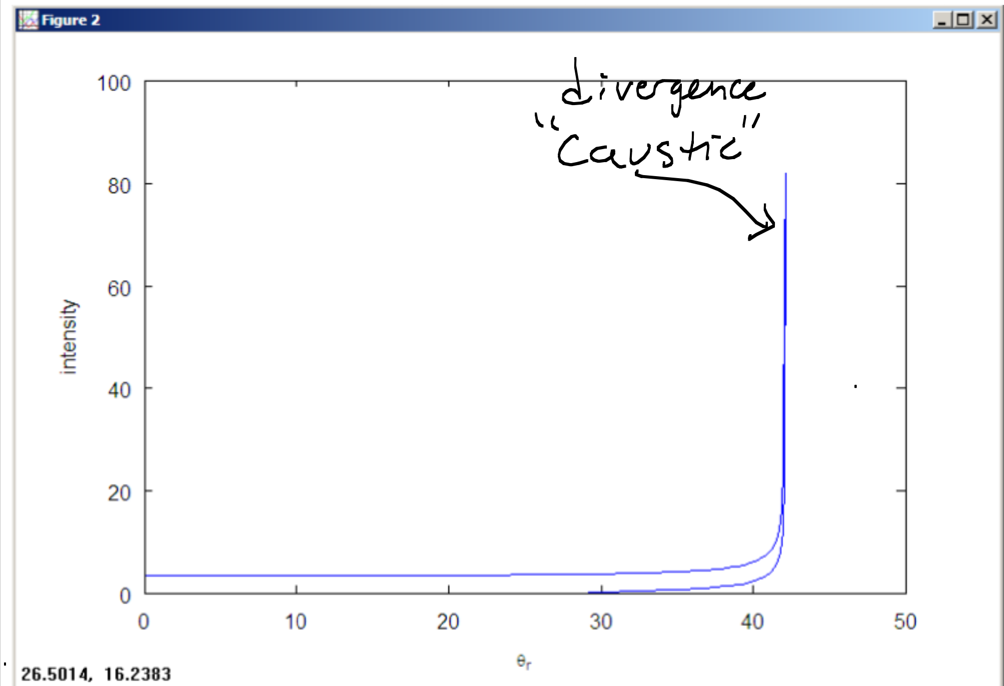
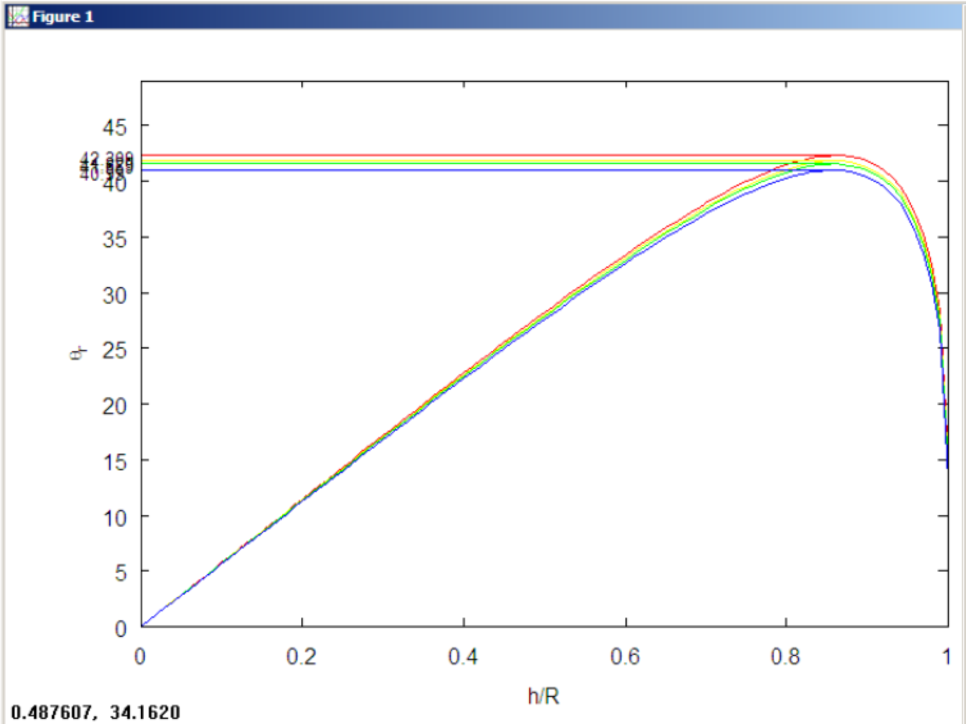
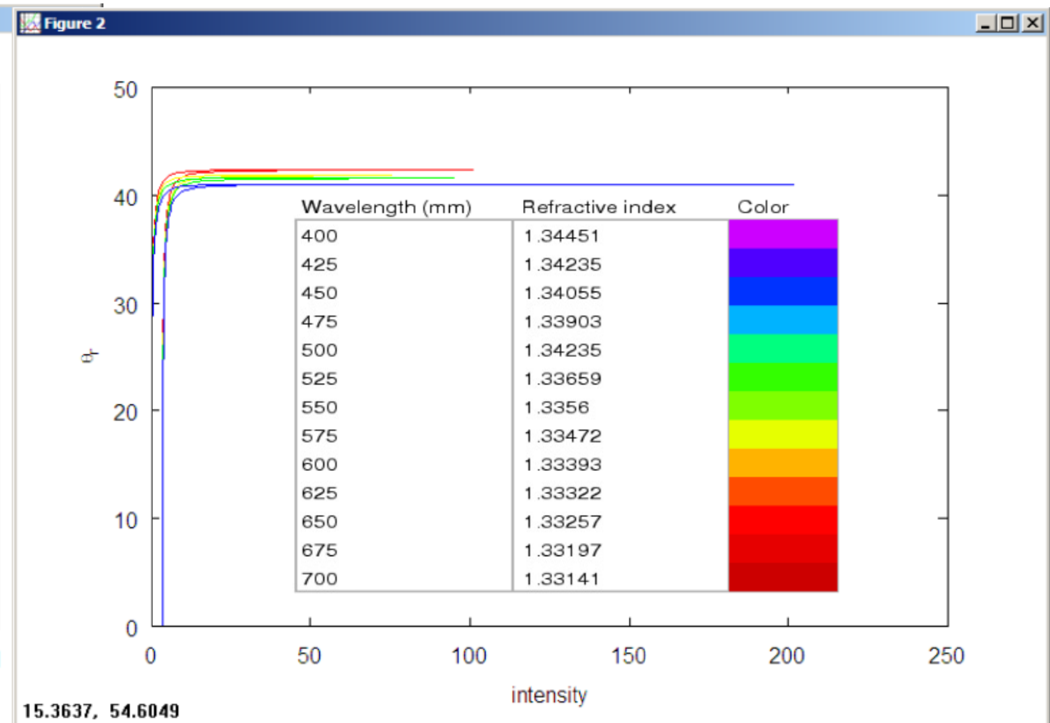


Figure 2
divergence
"caustic"
100
80
60
40
20
0
10
20
30
40
50
intensity
26.5014, 16.2383

Rainbows





Second rainbow from 2 internal reflections → colors reversed!

primary rainbow