Maxwell's eqn's: Integral form

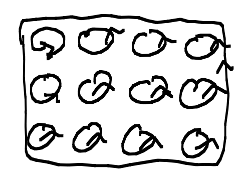
$$\oint\!\!\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$ "Gauss's Law"

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$ "Faraday's Law"

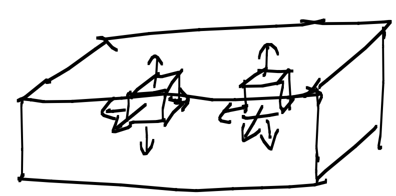
$$\oint\!\!\oint \vec{B} \cdot d\vec{S} = 0$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ "Ampere's Law"

"Stokes' Thm" $\oint \vec{A} \cdot d\vec{l} = \iint \vec{\nabla} \times \vec{A} \cdot d\vec{S}$



"divergence Thm" $\oint\!\!\oint \vec{A} \cdot d\vec{S} = \iiint \vec{\nabla} \cdot \vec{A}\, dV$



Differential form of Maxwell's Eqns.

$$\oiint \vec{E} \cdot d\vec{S} = \iiint \vec{\nabla} \cdot \vec{E} \, dV = \frac{Q}{\varepsilon_0} = \frac{\iiint \rho \, dV}{\varepsilon_0} \Rightarrow \boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$

$$\oint \vec{E} \cdot d\vec{l} = \iint \vec{\nabla} \times \vec{E} \cdot d\vec{S} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{S} \Rightarrow \boxed{\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}}$$

$$\oiint \vec{B} \cdot d\vec{S} = \iiint \vec{\nabla} \cdot \vec{B} \, dV = 0 \Rightarrow \boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

$$\oint \vec{B} \cdot d\vec{l} = \iint \vec{\nabla} \times \vec{B} \cdot dS = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \iint \left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right] \cdot d\vec{S}$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right)}$$

$\nabla \times$ (Faraday's Law)

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{d}{dt} \vec{\nabla} \times \vec{B} = -\frac{d}{dt} \left(\cancelto{0}{\mu_0 \vec{J}} + \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} \right)$$

0 in "free space" (no currents)

0 in "free space"

$$\nabla \left(\cancelto{0}{\nabla \cdot \vec{E}} \right) - \nabla^2 E = -\mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2}$$

(no charges)

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$[\mu_0 \varepsilon_0] = \frac{1}{\text{Velocity}^2}$$

$$\frac{1}{C^2} = \mu_0 \varepsilon_0 \qquad c \rightarrow \text{"speed of light"}$$

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{"wave equation"}$$

<u>1-D Solutions:</u> $F(z \pm ct)$ "D'Alembert solution"

"Simple" 1-D solution: "plane wave" $E_0 e^{i(kz \pm \omega t)}$

$$E_0 e^{ik(z \pm \frac{\omega}{k} t)}$$

$$\frac{\omega}{k} \equiv c$$

A more general solution allows variation transverse to propagation direction: "envelope"

$$\vec{E}(x, y, z, t) = \vec{A}(\vec{r})\, e^{i\omega t}$$

wave eqn:

$$\nabla^2 A = -\frac{\omega^2}{c^2} A = -k^2 A$$

$$\nabla^2 A + k^2 A = 0 \quad \text{"Helmholtz eqn"}$$

Plane-wave-like solution

$$A = U(\vec{r})e^{ikz}$$

$$\underset{x,y}{\nabla_T^2} u e^{ikz} + \frac{d}{dz}\left(\frac{du}{dz}e^{ikz} + u ik e^{ikz}\right) + k^2 A = 0$$

$$\nabla_T^2 u e^{ikz} + \frac{d^2u}{dz^2}e^{ikz} + \frac{du}{dz}ike^{ikz} + \frac{du}{dz}ike^{ikz} - k^2 u e^{ikz} + k^2 u e^{ikz} = 0$$

$$\nabla_T^2 u + 2ik\frac{du}{dz} + \cancel{\frac{d^2u}{dz^2}} = 0$$

neglect

$$\nabla_T^2 u + 2ik\frac{du}{dz} = 0$$

c.f. $\nabla^2\psi + i\hbar\frac{d\psi}{dt} = 0$

If $t \equiv z$, $\frac{\hbar^2}{2m} \equiv 1$, $\hbar \equiv 2k$ $\Longrightarrow$ like Schrödinger eqn. for free particle!

Solution

$$\frac{\partial^2}{\partial r^2} u + 2ik \frac{\partial u}{\partial z} = 0$$ (cylindrical symmetry $x \leftrightarrow y$)

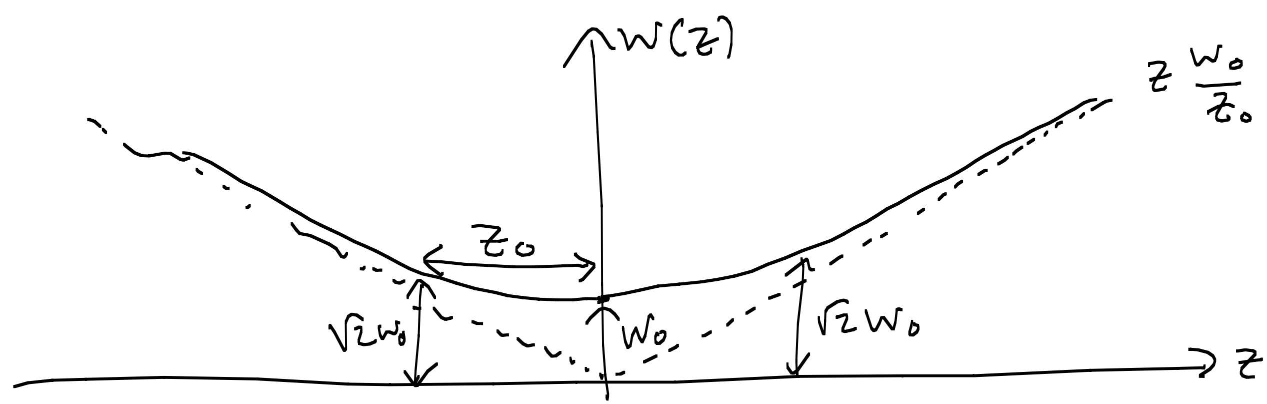
$$u = E_0 \, e^{i(\quad)} \, e^{-\frac{r^2}{w^2(z)}}$$

phase information

Since we measure Intensity $= |E|^2$

$$I \propto e^{-2r^2/w^2(z)}$$

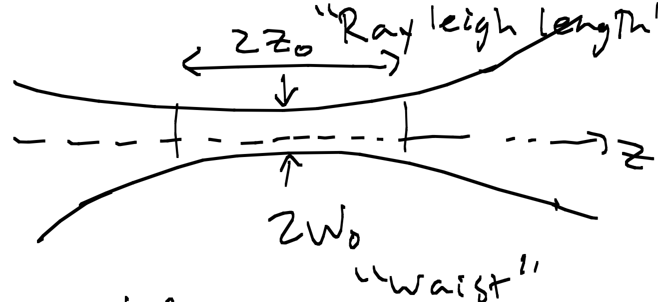
$$W(z) = w_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$



Uncertainty relation

$$\Delta r \, \Delta K > \sim \text{"}1\text{"}$$

depends on definition of uncertainty



Beam waist

$I = I_0 \, e^{-\frac{2r^2}{w_0^2}}$: Δr defined by $\frac{1}{e^2}$ point

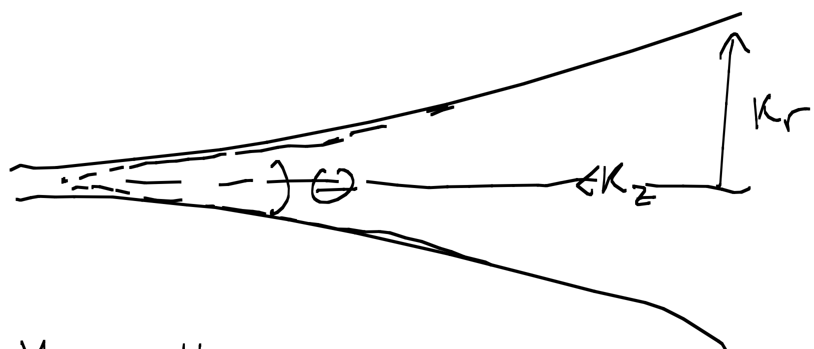
$$\Delta r = w_0$$

ΔK given by Fourier Transform

$$\propto e^{-\frac{K_r^2}{(8/w_0)}} \qquad \Delta K = \frac{4}{w_0}$$

$$K = \frac{2\pi}{\lambda}$$

$$= \sqrt{K_z^2 + K_r^2}$$



$$\Delta K = 2K_r = 2K \sin\frac{\theta}{2}$$

$$= K\theta$$

$$w_0 = \frac{4}{\Delta K} = \frac{4}{K\theta} = \frac{4\lambda}{2\pi\theta} = \frac{2\lambda}{\pi\theta}$$

Rayleigh length

$$K_z = K\cos\frac{\theta}{2} \sim K\left(1 - \frac{(\theta/2)^2}{2}\right) = K\left(1 - \frac{\theta^2}{8}\right)$$

$$\Rightarrow \Delta K = K\frac{\theta^2}{8}$$

By redefining "width"

$$\Delta z\, \Delta K_z = 2$$

$$\Delta z = 2z_0 = \frac{2}{\Delta K_z} = \frac{2}{K\theta^2/8}$$

$$\Rightarrow z_0 = \frac{8}{K\theta^2} = \frac{8\lambda}{2\pi\theta^2} = \frac{4\lambda}{\pi\theta^2}$$

$$= \frac{4\lambda}{\pi\left(\frac{2\lambda}{\pi w_0}\right)^2} = \frac{\pi w_0^2}{\lambda}$$

Lab 0