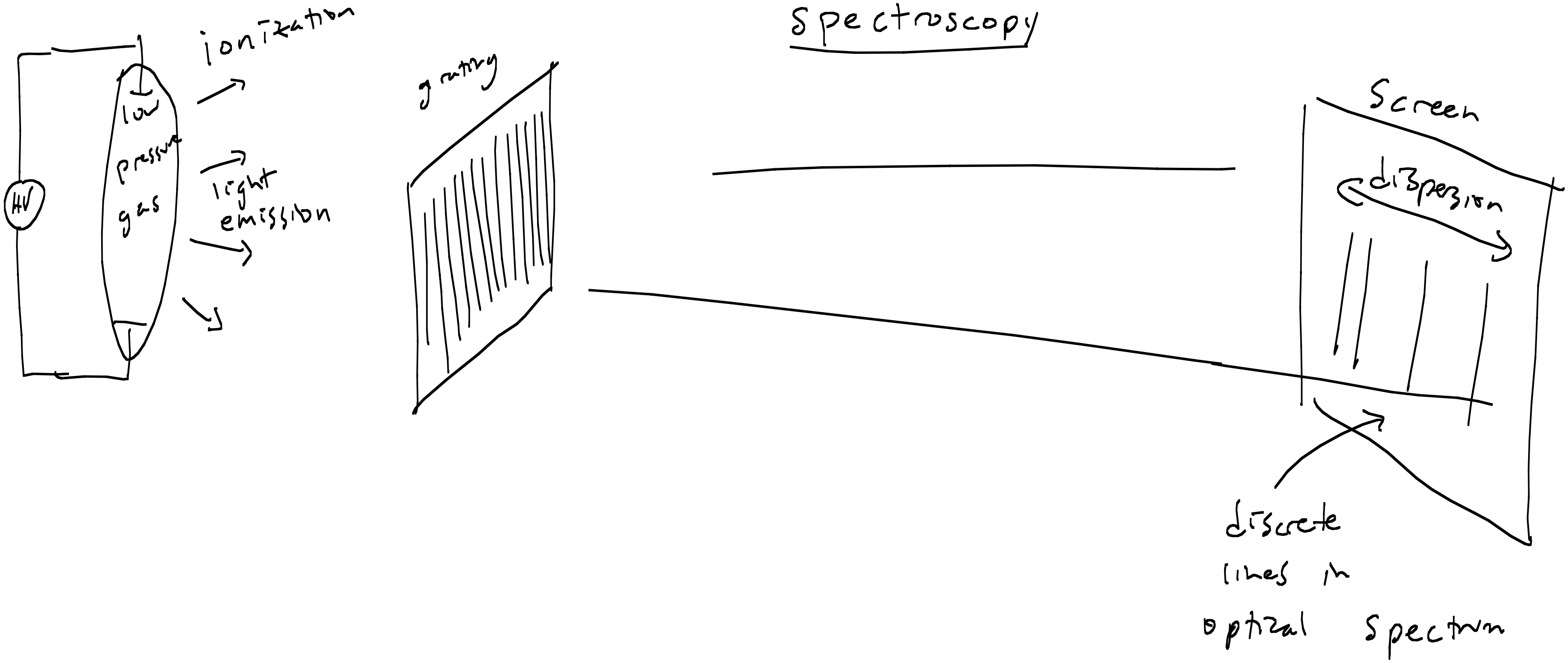
Atomic discharge

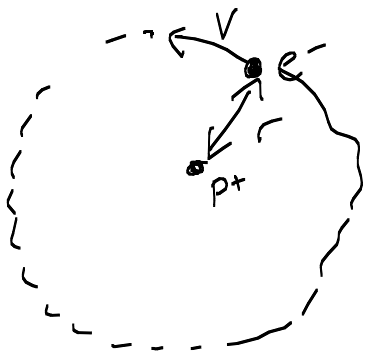


Bohr model

"quantized" angular momentum

① $mvr = n\hbar$ $\quad n = 1, 2, \ldots$

"principle quantum number"



Force balance

② $$\frac{mv^2}{r} = \frac{e^2}{r^2} \implies v = \sqrt{\frac{e^2}{mr}} \quad (*)$$

[centripetal = Coulomb]

subst. (*) into ①,

$$\sqrt{e^2 mr} = n\hbar \implies r = \frac{n^2\hbar^2}{e^2 m}$$

Total Energy = Kinetic + potential = $\frac{1}{2}mv^2 - \frac{e^2}{r}$

But, (*) gives Energy $= \frac{1}{2}\cancel{m}\frac{e^2}{\cancel{m}r} - \frac{e^2}{r} = -\frac{e^2}{2r}$

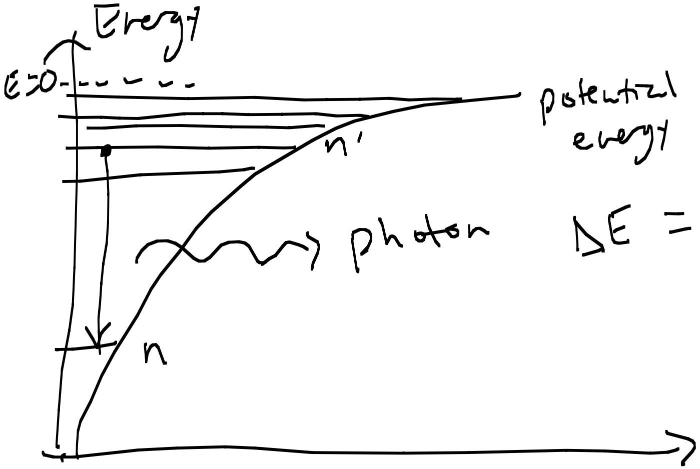
So, $\text{Energy} = -\frac{e^2}{2}\frac{e^2 m}{n^2\hbar^2} = \left(\frac{e^2}{\hbar c}\right)^2 \frac{mc^2}{2n^2}$

$\alpha \equiv \frac{e^2}{\hbar c}$ "fine structure constant" $\sim \frac{1}{137}$

$mc^2 \equiv E_0$ "rest mass" 511 keV

$\text{Energy} = -\alpha^2 \frac{E_0}{2n^2}$

"Radiative transitions"



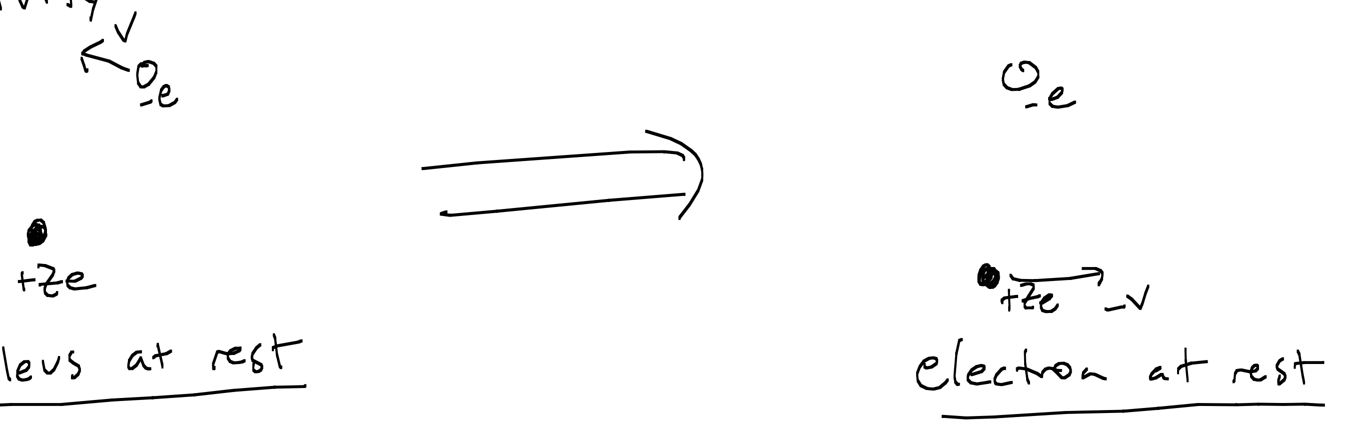
$\Delta E = -\frac{R}{n'^2} - \left(-\frac{R}{n^2}\right) = R\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$

↗

Rydberg (1888)
(13.6 eV)

Fine Structure: electron magnetic moment (spin)

relativity:



magnetic field at electron due to charge current from moving nucleus

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{Ze(-\vec{v}\times\vec{r})}{r^3} = -\frac{1}{4\pi\varepsilon_0 c^2}\frac{Ze\vec{L}}{mr^3}$$

$$(\vec{L} = m\vec{v}\times\vec{r})$$

$$\text{Energy} = -\vec{\mu}\cdot\vec{B} = -\Big(\overset{\text{electronic magnetic moment}}{g\frac{\mu_B}{\hbar}\vec{S}}\Big)\cdot\Big(\frac{-1}{4\pi\varepsilon_0 c^2}\frac{Ze\vec{L}}{mr^3}\Big) = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{m^2c^2r^3}\overset{\text{"spin-orbit"}}{\vec{S}\cdot\vec{L}}$$

Since $\vec{S}$ and $\vec{L}$ are in (near-unity) units of $\hbar$, we have

$\langle E_{\text{spin-orbit}} \rangle \sim \frac{e^2}{m^2c^2} \frac{\hbar \cdot \hbar}{r^3}$ and $\langle r^3 \rangle \sim \left(\frac{\hbar^2}{e^2 m}\right)^3$ so

$\langle E_{\text{spin-orbit}} \rangle \sim \frac{e^8 mc^2}{\hbar^4 c^4} = \left(\frac{e^2}{\hbar c}\right)^4 mc^2 \longrightarrow$ factor of α^2 smaller than electronic state splitting!

for fixed n, there are $2n-1$ possible values of L_z $(\vec{L} \cdot \hat{z}) \longrightarrow$ spin-orbit splitting of multiple states w/ same n.