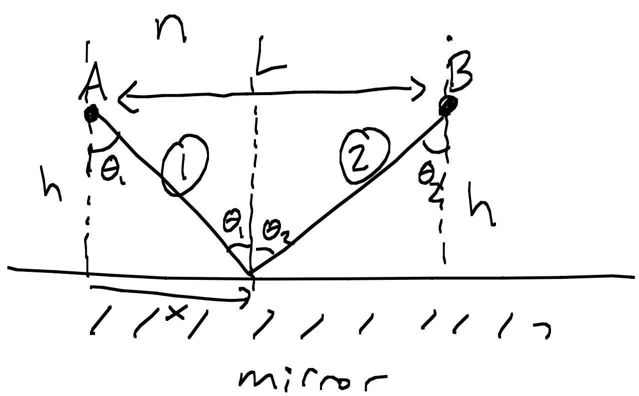
Fermat's Principle

Reflection:



time: ① + ②

$$\frac{\sqrt{h^2+x^2}}{c/n} + \frac{\sqrt{h^2+(L-x)^2}}{c/n}$$

$\frac{d}{dx}(\text{time}) = 0$:

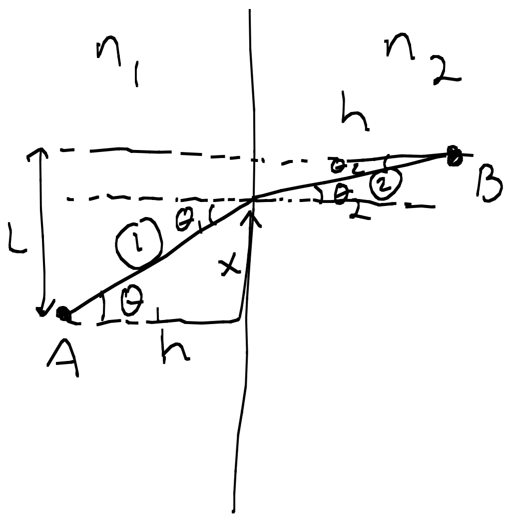
$$\frac{n}{c}\frac{1}{2}\frac{2x}{\sqrt{h^2+x^2}} + \frac{n}{c}\frac{1}{2}\frac{(-2)(L-x)}{\sqrt{h^2+(L-x)^2}} = 0$$

$$\frac{x}{\sqrt{h^2+x^2}} = \frac{L-x}{\sqrt{h^2+(L-x)^2}}$$

$$\sin\theta_1 = \sin\theta_2$$

$$\theta_1 = \theta_2$$

Refraction:



time: ① + ②

$$\frac{\sqrt{h^2+x^2}}{c/n_1} + \frac{\sqrt{h^2+(L-x)^2}}{c/n_2}$$

$\frac{d}{dx}(\text{time}) = 0$:

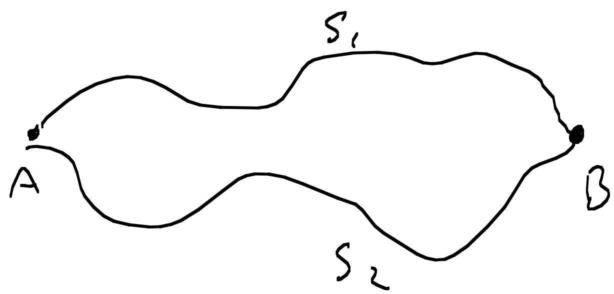
$$\frac{n_1}{c}\frac{1}{2}\frac{2x}{\sqrt{h^2+x^2}} - \frac{n_2}{c}\frac{1}{2}\frac{2(L-x)}{\sqrt{h^2+(L-x)^2}} = 0$$

$$n_1 \frac{x}{\sqrt{h^2+x^2}} = n_2 \frac{L-x}{\sqrt{h^2+(L-x)^2}}$$

"Snell's Law" $\quad n_1 \sin\theta_1 = n_2 \sin\theta_2$

Optical path length (OPL)

$$OPL = \int_A^B n(s)\,ds$$

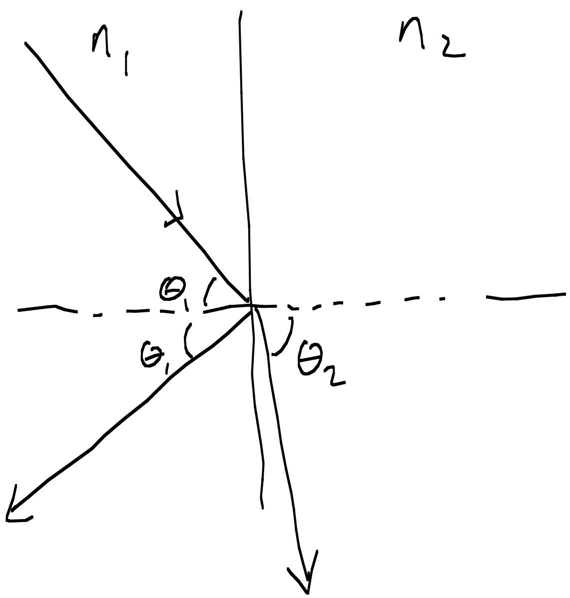


<u>or</u>

$$OPL = \sum_{j=1}^{m} n_j s_j$$

find local minima for $t = \frac{OPL}{c}$

total Internal reflection



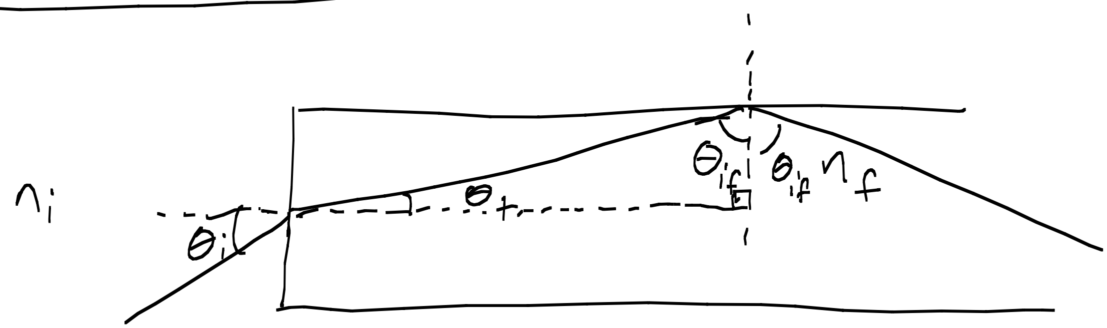
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 > n_2$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

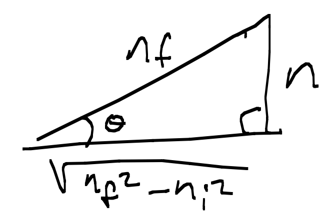
when $\frac{n_1}{n_2}\sin\theta_1 > 1$

Fiber optics



$$n_f \sin\theta_{if} = n_i \sin\frac{\pi}{2} = n_i \qquad \theta_{if} \geq \theta_{if}^c = \sin^{-1}\frac{n_i}{n_f}$$

$$\sin\theta_t = \cos\theta_{if}$$



Snell's Law: $$n_i \sin\theta_i^c = n_f \sin\theta_t = n_f \cos\theta_{if} = n_f \cos\left[\sin^{-1}\frac{n_i}{n_f}\right]$$

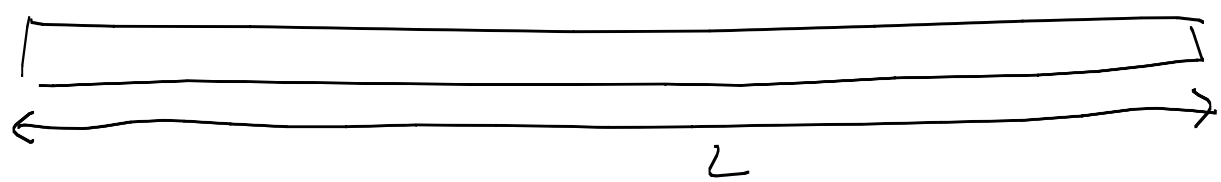
$$= n_f \frac{\sqrt{n_f^2 - n_i^2}}{n_f}$$

$$\theta_i^c = \sin^{-1}\frac{\sqrt{n_f^2 - n_i^2}}{n_i} = \sin^{-1}\left(\sqrt{\frac{n_f^2}{n_i^2} - 1}\right)$$

$$\theta_i < \theta_i^c$$

Intermodal dispersion

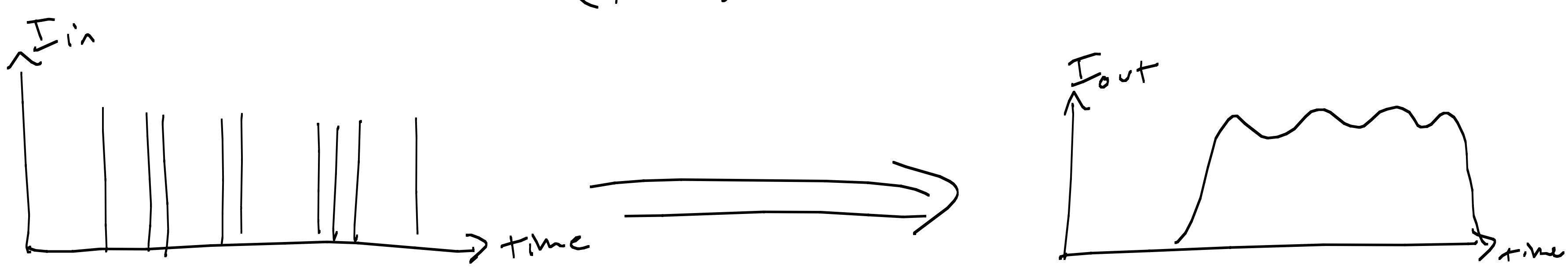
$$t = \frac{\text{length}}{\text{velocity}} \qquad \text{length} = \frac{L}{\cos\theta_t}$$



$$n_i \sin\theta_i = n_f \sin\theta_t \implies \sin\theta_t = \frac{n_i}{n_f}\sin\theta_i$$

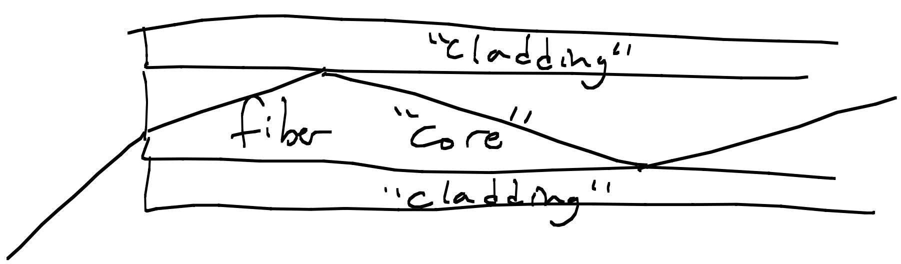
$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t}$$

$$\text{length} = \frac{L}{\sqrt{1 - \left(\frac{n_i}{n_f}\sin\theta_i\right)^2}}$$



Solutions:

1. Reduce critical incident angle



$n_{cladding} \sim< n_{core}$

2. GRIN graded index