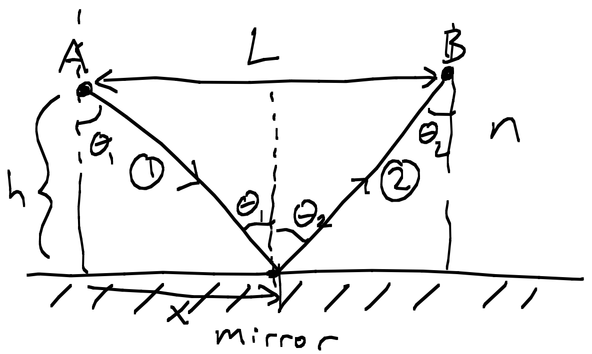
Fermat's Principle

Reflection:



$$\text{time: } \overset{①}{\frac{\sqrt{h^2+x^2}}{c/n}} + \overset{②}{\frac{\sqrt{h^2+(L-x)^2}}{c/n}}$$

$$\frac{d}{dx}(\text{time}) = 0 :$$

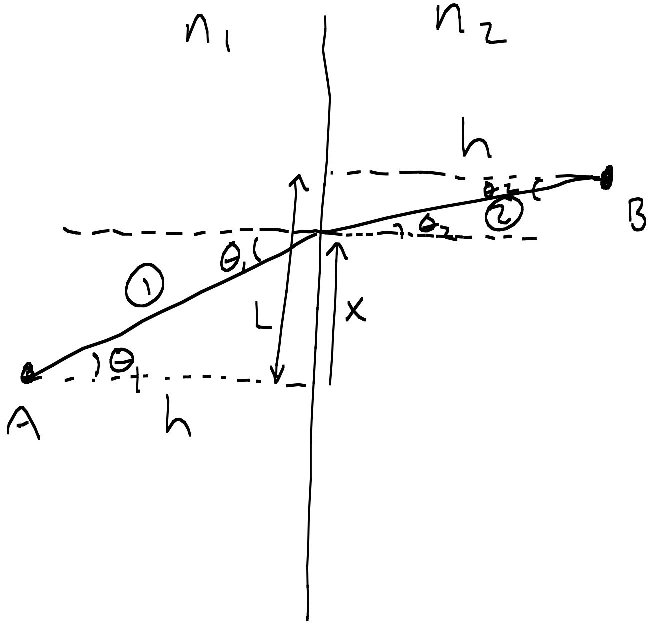
$$\frac{n}{c}\,\frac{1}{2}\,\frac{2x}{\sqrt{h^2+x^2}} + \frac{n}{c}\,\frac{1}{2}\,\frac{-2(L-x)}{\sqrt{h^2+(L-x)^2}} = 0$$

$$\frac{x}{\sqrt{h^2+x^2}} = \frac{L-x}{\sqrt{h^2+(L-x)^2}}$$

$$\sin\theta_1 = \sin\theta_2$$

$$\theta_1 = \theta_2$$

Refraction:



minimize time!

$$\text{time:}\quad \overset{①}{\frac{\sqrt{h^2+x^2}}{c/n_1}} + \overset{②}{\frac{\sqrt{h^2+(L-x)^2}}{c/n_2}}$$

$$\frac{d}{dx}(\text{time}) = 0 :$$

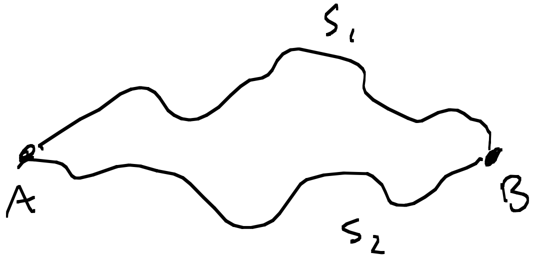
$$\frac{n_1}{c}\frac{1}{2}\frac{2x}{\sqrt{h^2+x^2}} + \frac{n_2}{c}\frac{1}{2}\frac{-2(L-x)}{\sqrt{h^2+(L-x)^2}} = 0$$

$$n_1\frac{x}{\sqrt{h^2+x^2}} = n_2\frac{L-x}{\sqrt{h^2+(L-x)^2}}$$

"Snell's Law" $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Optical path length

$$OPL = \int_A^B n(s)\,ds$$



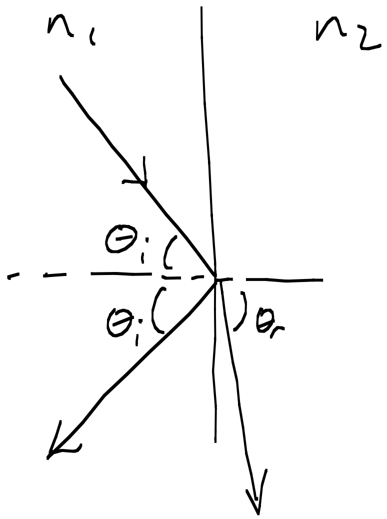
or

$$OPL = \sum_{j=1}^{m} n_j s_j$$

$$t = \frac{OPL}{c} = \frac{1}{c}\sum_{j=1}^{m} n_j s_j$$

find local minima for t

Total internal reflection



$$n_1 \sin\theta_i = n_2 \sin\theta_r$$

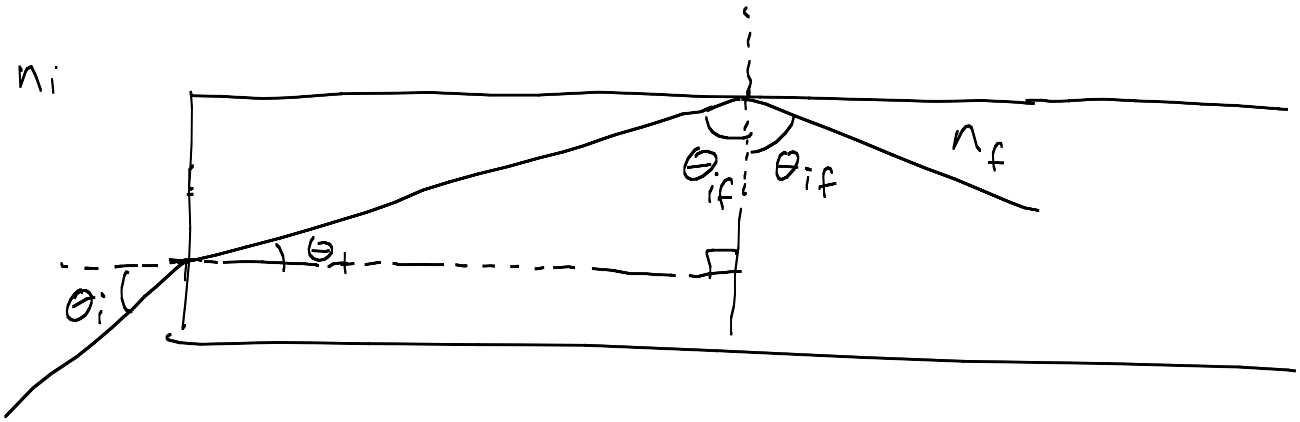
$$\theta_r = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_i\right)$$

If $\frac{n_1}{n_2} > 1$

when $\frac{n_1}{n_2}\sin\theta_i > 1$, total internal reflection occurs

(No refracted ray!)

Fiber optics

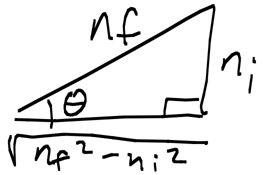


$$n_f \sin\theta_{if} = n_i \sin \pi/2 = n_i \implies \theta_{if} > \theta_{if}^c = \sin^{-1}\frac{n_i}{n_f}$$

$$\sin\theta_t = \cos\theta_{if}$$

Snell's Law $n_i \sin\theta_i^c = n_f \sin\theta_t = n_f \cos\theta_{if} = n_f \cos\left[\sin^{-1}\frac{n_i}{n_f}\right]$

$$\theta_i^c = \sin^{-1}\left[\frac{n_f}{n_i}\cos\left(\sin^{-1}\frac{n_i}{n_f}\right)\right]$$

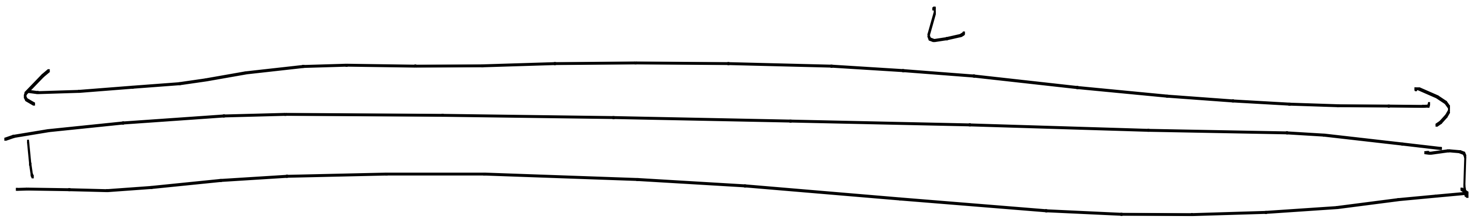


$$\theta_i^c = \sin^{-1}\left[\frac{n_f}{n_i}\frac{\sqrt{n_f^2-n_i^2}}{n_f}\right] = \sin^{-1}\left[\sqrt{\left(\frac{n_f}{n_i}\right)^2-1}\right]$$

$$\theta_i < \sin^{-1}\left[\sqrt{\left(\frac{n_f}{n_i}\right)^2-1}\right]$$ max. angle of incidence for TIR

Intermodal dispersion

$$t = \frac{\text{length}}{\text{velocity}} \qquad \text{length} = \frac{L}{\cos\theta_t}$$



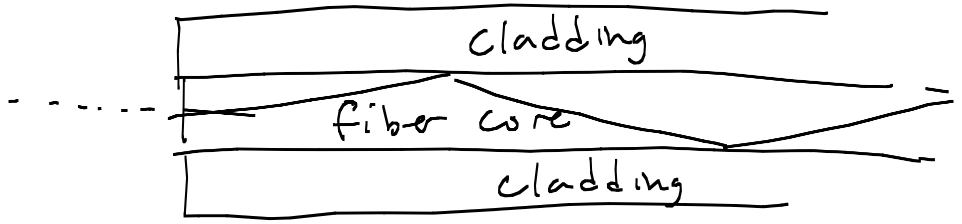
Snell's law $\quad n_i \sin\theta_i = n_f \sin\theta_t \Rightarrow \sin\theta_t = \frac{n_i}{n_f}\sin\theta_i$

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t}$$

$$\text{length} = \frac{L}{\sqrt{1-\left(\frac{n_i}{n_f}\right)^2\sin^2\theta_i}} = \frac{L n_f}{\sqrt{n_f^2 - n_i^2\sin^2\theta_i}}$$

Solutions:

1. reduce critical incident angle w/ cladding



$n_{cladding} \lesssim n_{core}$

2. GRIN: graded index fiber

slows down normal-incidence rays