Experiment 6

Study of Atomic Spectra

1 Introduction

In this experiment, we will employ a plane diffraction grating to create a spectrometer, which we will use to measure a few atomic emission lines of hydrogen (H), sodium (Na) and mercury (Hg). We will use the Hg lines to calibrate the spectrometer. The goals of this experiment are (1) to use the H spectrum to estimate the Rydberg constant and (2) to determine the energy level diagram of the low lying levels of Na and the fine structure splitting of the $3p$ configuration.

2 Background- *see Pedrotti^3, Section 6-1 and Chap. 12*

2.1 Diffraction Grating

In this experiment, we will use a plane diffraction grating, the large N limit of multiple slits. Diffraction gratings can be made by ruling a large number of equidistant lines on a glass substrate. Most gratings today, however, are replicas (molds of ruled gratings).

The maxima produced by a diffraction grating are very sharp. For parallel light incident on a diffraction grating with angle θ_i, the positions of the maxima are given by

$$p\lambda = d\left(\sin\theta_p + \sin\theta_i\right), \qquad p = \pm 1, \pm 2, \pm 3, \ldots \qquad (1)$$

where p, d, θ_p and θ_i are defined in Fig. 1 with p designating the order of the spectrum.

It is clear from Eq. (1) that for fixed p, the angle θ_p will be a function of the wavelength of the light. Thus, if we illuminate the grating with light composed of several wavelengths, each wavelength will emerge with a different angle. Note, however, dispersion only occurs for $|p| \geq 1$. At zero order, $\theta_p = 0$, we will see all the wavelengths.
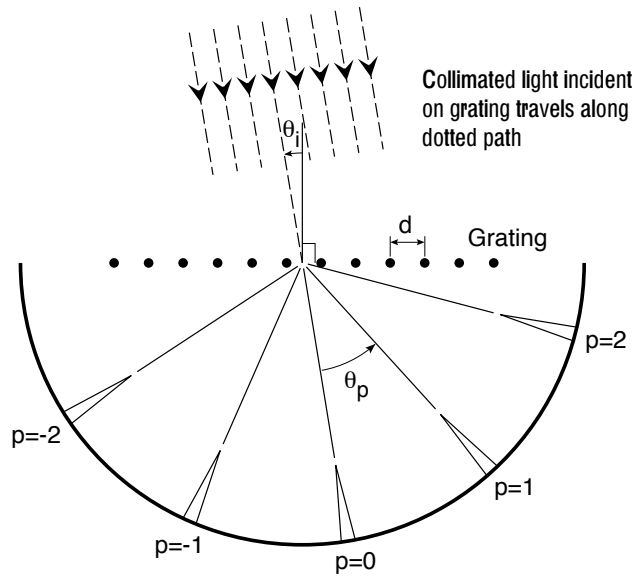
Figure 1: Schematic illustration of diffraction of light by a diffraction grating. Various orders of the spectrum are shown.

The diffraction grating is a particularly simple instrument to use for studying the components of light produced by an excited atom. When the electrons in an atom are excited (have their energy increased), they return to lower states by emitting light of specific wavelengths known as spectral lines. The spectrum of an atom is one of its basic signatures. The existence of many elements in astronomical studies is often established by measurements of their spectra. Atomic spectroscopy has also played a fundamental role in the development of quantum mechanics. Hence, the great theoretical and experimental importance of atomic spectroscopy is well established.

2.2 Hydrogen Atom

In 1889, Rydberg proposed the following formula to describe the frequency of the light emitted when a Hydrogen atom makes a transition between energy levels n and n',

$$\tilde{\nu}_{n'n} = \frac{E_{n'} - E_n}{hc} = \mathcal{R}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right). \tag{2}$$

In Eq. (2), $\tilde{\nu}(\equiv \nu/c)$ is called the wavenumber – the frequency of the light divided by the speed of light and expressed in units of cm^{-1} – E_n is energy of level n, hc is Planck's constant times the speed of light and the constant $\mathcal{R}$ is the Rydberg constant.

For H, the energy of the nth level given by

$$E_n = -\frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon_o n^2 a_o}\right), \tag{3}$$

where a_o is the Bohr radius. From Eqs. (2) and (3) it follows that the Rydberg constant for an infinitely massive nucleus is given by

$$\mathcal{R}_\infty = \frac{1}{2hc}\left(\frac{e^2}{4\pi\varepsilon_o a_o}\right) = \frac{m_e e^4}{4\pi c\hbar^3}, \tag{4}$$

while the Rydberg constant for an atom of mass M is given by

$$\mathcal{R}_M = \mathcal{R}_\infty\left(1 + \frac{m_e}{M_N}\right)^{-1}. \tag{5}$$

The energy unit of $\mathcal{R}$ is most often cm^{-1}. Combining Eqs. (2) and (4) leads to the following simple result for the H spectrum:

$$E_n = -hc\mathcal{R}_H\frac{1}{n^2}. \tag{6}$$

2.3 Sodium Atom

The structure of Na is more complicated than that of H because of the presence of 10 additional electrons. Although these electrons are tightly bound in the $1s$, $2s$ and $2p$ levels, they produce a significant affect on the energy of the outer electron in the $3p$ level. The predominant lines in the Na spectrum are from transitions from the excited $3p$ level to the $3s$ ground state. For the $3p$ level of Na, $n = 3$, $l = 1$ and $s = 1/2$ leading to total angular momentum $j = 1/2$ and $j = 3/2$. The energy difference of these two levels is known as fine structure, and is due to the spin-orbit coupling, the interaction of the orbital angular momentum with the spin angular momentum. These yellow lines are the same as those measured previously with the Michelson interferometer.

2.4 Mercury Atom

The structure and spectra of the Hg is very complicated, due to the large number of electrons ($Z = 80$). Mercury is added to fluorescent lights because the large number of different wavelength lines in the spectra yields an overall color close to white. We will use mercury lines to calibrate the spectrometer.

3 Experiment

3.1 Overview of the Spectrometer

The apparatus used in this experiment is schematically shown in Fig. 2. It consists of a plane transmission diffraction grating (~ 600 lines/mm) mounted on the rotating table of a spectrometer. The spectrometer also consists of a telescope and a collimator, both of which can be rotated independently about the axis of the table. A description of each of the numbered parts will now be given.
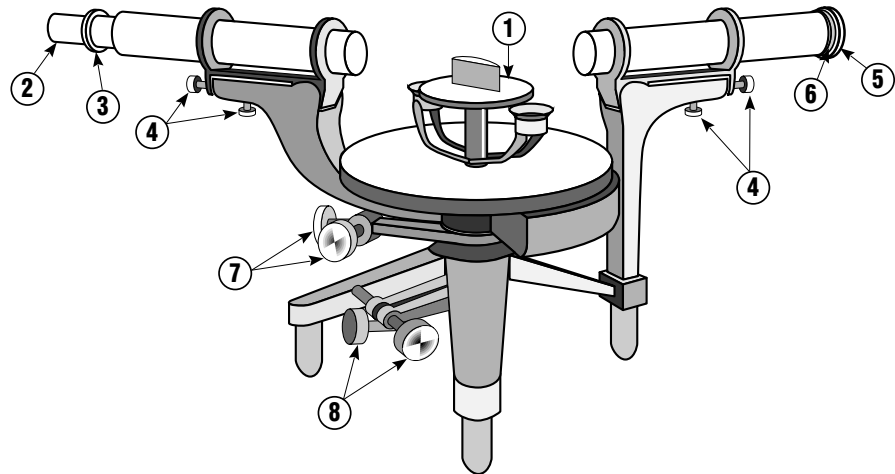
Figure 2: The experimental apparatus used to study atomic spectra.

1. **Table**. Used for mounting the grating.

2. **Eyepiece focus**. At the end of the telescope, this eyepiece can be gently pulled or pushed to get a sharp focus on the crosshairs without moving their position. This allows the apparatus to allow for different individual viewing preferences without upsetting the focus of the image being viewed.

3. **Telescope focus**. The chrome flange can be rotated to focus the telescope on the image.

4. **Leveling Screws**. These are used to align the collimator and telescope vertically . One screw is a clamp and the other is used for leveling.

5. **Collimator stop**. Adjusts the length of the collimator. The collimator should be set to the correct length when the stop is pushed in against the outer shell.

6. **Collimator slit** Allows adjustment in width and orientation of the slit.

7. **Telescope arm clamping and tangent screws**. The two screws on the side of the spectrometer just below the telescope each serve a distinct purpose. The screw projecting radially outward is the clamping screw which should always be loosened before attempting to move the telescope. After the telescope is nearly in the desired position, the clamping screw should be gently tightened. The perpendicular tangent screw then allows for very fine adjustment of the telescope position.

8. **Object table clamping and tangent screws**. These adjust the rotating table in the same way as (7) adjusted the position of the telescope arm. They should be used in the same way.

3.2 Aligning the Spectrometer

First, focus the telescope for parallel light (i.e., light from infinity) by focusing on an object several tens of meters away. Focusing through a window on a building across the street will do. Then, minimize the parallax between the focused image and the cross-hairs. That is, locate the image in the plane of the cross-hairs.

Second, put a light source close to the slit of the collimator. Using the telescope, view the slit. Make the slit as narrow as possible. While looking through the telescope, adjust the focus of the collimator so that the image of the slit falls on the plane of the cross-hairs. This is done by removing any parallax between the image of the slit and the cross-hairs. Now the collimator is set to produce a collimated beam, i.e., a parallel beam of light.

Third, mount the grating on the table of the spectrometer. Make sure that the photographic film from which the grating is made is on the telescope side.   (The excess glass should protrude towards the telescope side.)   Note the position of the telescope when it is in line with the collimator, i.e., when the slit image is aligned with the cross-hairs. Shine the light from your desk lamp into the open side port of the telescope (near the eyepiece) to illuminate the cross-hairs. You might find it helpful to place a mirror just behind and flush with the grating to see the reflected image of the cross-hairs. Once you have located it, you can remove the mirror and continue with the alignment procedure. Rotate the grating so that the reflected image of the cross-hairs falls back on the cross-hairs. In this position, the grating is perpindicular to the optical axis of the telescope. The system is set up to approximately satisfy Eq. (1); the incident beam consists of parallel light falling nearly normally on the grating, i.e., $\theta_i = 0$.   Lock the grating into place and record the position of the angular scale on the rotating table. Do not adjust the grating anymore.   You will be able to see images of successive orders of the spectrum by rotating the telescope away from the zero order position.

3.3 Making Measurements of Various Spectra

As mentioned earlier, the sources you will be observing are mercury, hydrogen, and sodium. You will be able to see first order spectra for all of these light sources. With the room lights out, you will also be able to detect some of the lines in second order.

To make measurements, first record the value of the angular scale on

the rotating table when the cross-hairs are aligned with the zero order spectrum (this corresponds to θ_0). Rotate the telescope to the right so that the cross-hairs align with a pth order spectral line. Record the angular scale reading. Then, rotate the telescope so that the cross-hairs align with the corresponding spectral line on the left side of the zero order spectrum; record the angular scale reading. You should measure the angle for each spectral line at least twice on each side, giving you at least four measurements of the angle, θ_p for the pth order. Take the average as your result and the standard deviation as your uncertainty. This should be done for each visible spectral line. Note, you might find it easier to measure all the lines on one side before measuring them on the other side. With these measurements, you will be able to calculate λ, the wavelength of each spectral line.

The scale on the spectrometer is a vernier scale. The outside scale is ruled in 0.5 degree; the inside scale is ruled in minutes of arc. To read the angle, look at 0 on the inner scale. Say it falls between 180.0 and 180.5 on the outer scale. Now look for the point where the outer and inner scales line up. Say it is aligned with 24 on the inner scale. Your angle would be 180° 24'. If the 0 of the inner scale were between 180.5 and 181.0 then your angle would have been 180° 54'.

Make sure you understand how to read a vernier scale:

http://www.phys.hawaii.edu/%7Eteb/java/ntnujava/ruler/vernier.html

When making your measurements, <u>always</u> rotate the table in the same direction, cw or ccw, to minimize errors due to hysteresis.

3.3.1 The Mercury Spectrum

You will use the Hg spectrum as your wavelength standards to calibrate your spectrometer. The wavelengths can be on the National Institute of Standards and Technology atomic database website: (http://physics.nist.gov/PhysRefData/ASD/index.html). You should measure the angles of the five lines (violet, blue, blue-green, green and yellow). From the known wavelengths, you can use Eq. (1) to refine your value of d ($\sim 1/600$ mm). Do this by fitting λ to $d\sin\theta$ and extracting a value for d from the fit parameters.

3.3.2 The Hydrogen Spectrum

With the hydrogen lamp, measure the wavelengths of the violet, blue and red spectral lines. The wavelengths you measure are the air wavelengths. To determine the Rydberg constant, you will need vacuum wavelengths,

$$\lambda_{vac} = 2\pi c/\omega = 1/\tilde{\nu}, \tag{7}$$

where ω is the angular frequency. Given the air wavelength, the vac-

uum wavelength can be calculated with

$$\lambda_{vac} = n_{air}\lambda_{air}, \tag{8}$$

where n_{air} is the index of refraction of air; its value can be found in the Handbook of Physics and Chemistry or on the web. You can determine $\mathcal{R}_H$ by fitting $1/\lambda_{vac}$ to

$$\mathcal{R}_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right). \tag{9}$$

Convert $\mathcal{R}_H$ to $\mathcal{R}_\infty$ and compare your estimate with the tabulated value.

3.3.3 The Sodium Spectrum

With the sodium lamp, you should measure the wavelengths of the green, yellow and red spectral lines in first order, and compare to the database values.

You should also measure the yellow Na lines in 2nd order. Calculate the splitting of the yellow doublet in wavenumbers, and compare to the database values.