Problem 1:

(a)

Mean:
```
function ret=mymean(a)
  ret=sum(a)/length(a);
endfunction
```

Mode:
```
function ret=mymode (a)
  z=zeros(1, length(a));
  for ii=1:length(a)
    for jj=1:length(a)
      if ((a(jj)-a(ii))==0)
        z(ii)=z(ii)+1;
      end
    end
  end
  [w,iw]=max(z);
  ret=a(iw);
endfunction
```

Median:
```
function ret=mymedian(a)
  asort=sort(a);
  N=length(a);
  if ceil(N/2)==floor(N/2)
    ret=(asort(N/2)+asort(N/2+1))/2;
  else
    ret=asort(ceil(N/2));
  end
endfunction
```

Variance:
```
function ret=myvariance(a)
  ret=  sum((a-mymean(a)).^2)/(length(a)-1);
endfunction
```

Standard Deviation:
```
function ret=mystdev(a)
  ret=sqrt(myvariance(a));
endfunction
```

a=[0.91595 0.35290 0.35692 0.18598 0.67537 0.92017 0.98268 0.44933 0.27089 0.81826];
mymean(a)
 ans = 0.59284
mymode(a)
 ans = 0.91595
mymedian(a)
 ans = 0.56235

mystdev(a)
 ans = 0.30261
myvariance(a)
 ans = 0.091573

a1000=rand(1000,1);
mymean(a1000)
 ans = 0.50032
mymode(a1000)
 ans = 0.19736
mymedian(a1000)
 ans = 0.50370
mystdev(a1000)
 ans = 0.28774
myvariance(a1000)
 ans = 0.082792

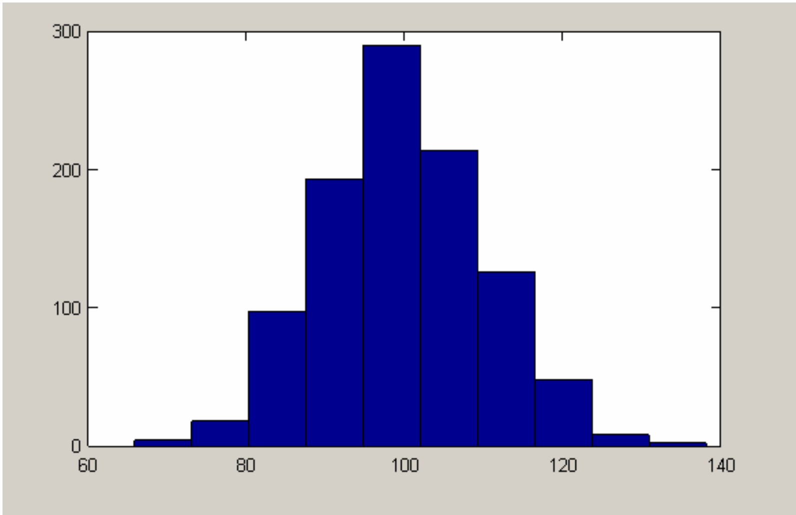
(b)

Generate random numbers from Gaussian PDF:

```
function ret=mygaussian(xmean, xsigma)
  ret=[];
  while (isempty(ret))
    x=xmean+(0.5-rand)*8*xsigma; %4 sigma range
    if rand<(e^(-((x-xmean)^2)/(2*xsigma^2)))
      ret=x;
    end
  end
endfunction
```

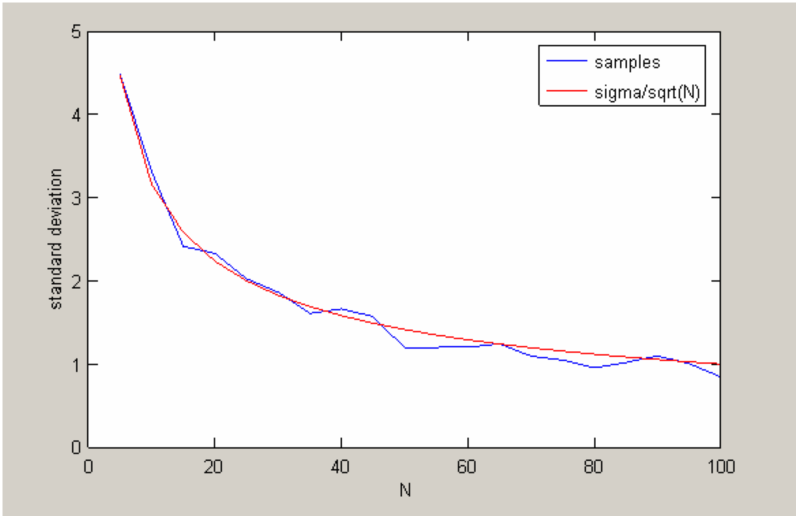
Show that this works:

```
for ii=1:1000
  g(ii)=mygaussian(100,10);
end
hist(g)
```



Demonstrate that the standard deviation of the mean tends to zero as the sample size increases:

```
clear;
xmean=100;
xsigma=10;
Ns=5:5:100;
kk=1;
for N=Ns
  jj=1;
  for samples=1:25 %num. of samples to calculate stdev from
    for ii=1:N
      a(ii)=mygaussian(xmean,xsigma);
    end
    m(jj)=mymean(a);
    jj=jj+1;
  end
  s(kk)=mystdev(m);
  kk=kk+1;
end
plot(Ns,s,'b;samples;');
hold on;
plot(Ns,xsigma./sqrt(Ns),'r;sigma/sqrt(N);');
hold off;
xlabel('N');
ylabel('standard deviation');
```

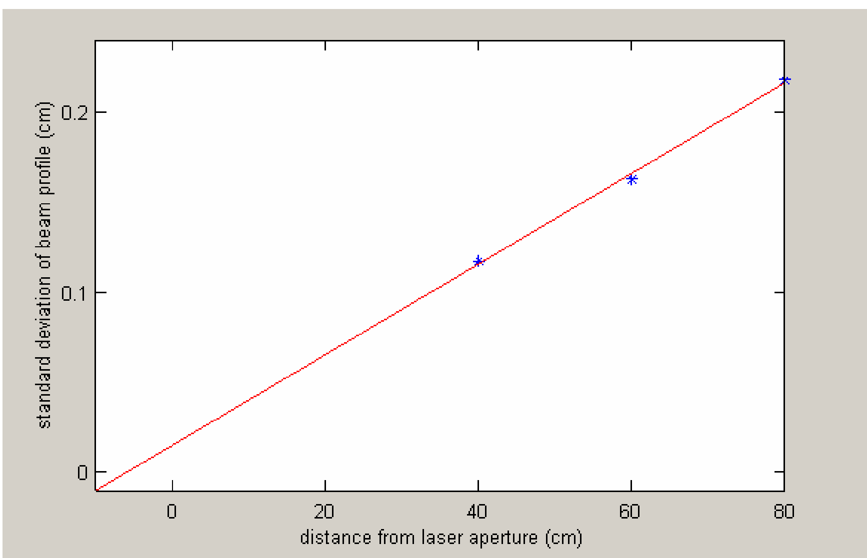


Note that analytic expression closely matches sampled standard deviation of the mean!

<u>(c)</u>

Mean and Standard Deviation:

```
mn=data(:,2)'*data(:,1)/sum(data(:,2))
sd=sqrt((data(:,2)'*(data(:,1)-mn).^2)/sum(data(:,2)))
```



If $I = I_0 e^{-\frac{2r^2}{w^2}}$, and Gaussian form is $I = I_0 e^{-\frac{r^2}{2\sigma^2}}$, then $w = 2\sigma$.
The divergence angle θ is then given by 2*atan(2*slope). Reasonable values are in the range 2x10^{-3} rad (~0.1 degree).

Waist position is given by the x-intercept beam waist $w_0 = \frac{2\lambda}{\pi\theta}$ (should be on the order of 100μm)

Rayleigh length $z_0 = \frac{\pi w_0{}^2}{\lambda}$ (should be on the order of 10cm)

2. **Error propagation**. You are trying to determine the acceleration due to gravity *g* by measuring the period of a pendulum, *T*, of length *L* using the relation $T = 2\pi\sqrt{\frac{L}{g}}$.
The summary of measured data is $T = 3.818 \pm 0.009$ *sec*, and $L = 361.58 \pm 0.40$ *cm*. By propagating errors, determine the best value and uncertainty in *g*. If you could go back and revise the experiment, which quantity would you want to measure more precisely?

Rearrange the above equation to give (1) $g = 4\pi^2 \frac{L}{T^2}$. *Using the error propagation formula we find (2)* $\frac{\sigma_g}{g} = \sqrt{\left(\frac{\sigma_L}{L}\right)^2 + \left(\frac{\sigma_{T^2}}{T^2}\right)^2} = \sqrt{\left(\frac{\sigma_L}{L}\right)^2 + \left(2\frac{\sigma_T}{T}\right)^2}$. *The best value of g is given by (1)* $g = 4\pi^2 \frac{361.68cm}{(3.818\,\text{sec})^2}$ = 979.2 cm/s^2. *Plugging into (2) we find*

$$\frac{\sigma_g}{g} = \sqrt{\left(\frac{0.40}{361.8}\right)^2 + \left(2\frac{0.009}{3.818}\right)^2}$$ = 0.00484 and σ_g = 4.7 cm/s^2. *In the previous formula the ratio of the length uncertainty to the length is small so the error is dominated by the uncertainty in the period.*

3. Pedrotti[3], 3rd edition, problem 1-3.

*The energy of a photon is given by $E = h\nu = hc/\lambda$. Thus at λ = 380 nm and 770 nm, E= $(6.63\text{x}10^{-34}$ J*s)$(3\text{x}10^{8}$m/s)/$(380\text{x}10^{-9}$ m) = $5.23\text{x}10^{-19}$J and $(6.63\text{x}10^{-34}$ J*s)$(3\text{x}10^{8}$m/s)/$(770\text{x}10^{-9}$ m) = $2.58\text{x}10^{-19}$J. Since 1eV = $1.6\text{x}10^{-19}$ J the energies are 3.27 eV and 1.61 eV respectively.*

4. Pedrotti[3], 3rd edition, problem 1-15.

(a) $t = D_l/c = (90\text{x}10^3)/3.0\text{x}10^8$s = 3.0 x 10^4s
(b) $D_s = v_s t$ = (340)$(3.0\text{x}10^{-4}$m) = 0.10 m

5. An electromagnetic wave is specified (in SI units) by the following function:

$$\vec{E} = (-6\hat{i} + 3\sqrt{5}\hat{j})(10^4 V/m)\exp[i\{\frac{1}{3}(\sqrt{5}x + 2y)\pi \times 10^7 - 9.42 \times 10^{15} t\}]$$

Find (a) the direction along which the electric field oscillates,

The field oscillates in $(-6\hat{i} + 3\sqrt{5}\hat{j})$ direction. Normalizing this gives the unit vector $(-\frac{2}{3}\hat{i} + \frac{\sqrt{5}}{3}\hat{j})$. One could also find an angle direction wrt to the x-axis using tan θ = $\sqrt{5}/2$ = 48.2+90 = 138.2 degrees

(b) the scalar value of amplitude of electric field,

Take the dot product of the amplitude and then the square root. $\sqrt{36+45}$ x 10^4 V/m = 9 x 10^4 V/m

(c) the direction of propagation of the wave,

Since the exponential is $\vec{k}\cdot\vec{r} - \omega t$, the wave travels in the direction of $\vec{k}$.

(d) the propagation number and wavelength, *The dot product of* $\vec{k}$ *and* $\vec{r}$ *in the exponential is* $\{\frac{1}{3}(\sqrt{5}x + 2y)\pi \times 10^7$ which *implies* $\vec{k} = \sqrt{5}\hat{i} + 2\hat{j}(\pi/3)10^7$. *Then* $\vec{k} \cdot \vec{k} = (\pi x 10^7)^2$ so $k = \pi x 10^7 m^{-1}$. *Finally,* $\lambda = \frac{2\pi}{k} = 200nm$.

(e) the frequency and angular frequency, and

ω = 9.42x10^{15} *rad/s and ν = ω/2π = 1.5 x 10*^{15} *Hz*

(f) the speed.

v= *νλ = 3x10*^{8} *m/s, the speed of light.*

6. An underwater swimmer shines a beam of light up toward the surface. It strikes the air-water interface at 35°. At what angle will it emerge into the air?

The index of refraction of water is 1.33. Using Snell's Law we get 1.33 sin 35 = 1.00 sin θ_t. *Solving,* $\theta_t = 50$ *degrees.*