# Polarization Review
# Phys 375

**See Pedrotti[3], Section 4-9, Chapter 14, and Sections 15-1 to 15-3**
**Polarization**

The polarization state of a plane wave is specified by the direction of the electric field vector. The polarization state can be linear, elliptical, or circular. A general expression for the E-field is:

$$\vec{E} = \hat{\imath}\, E_{0x}\, cos(kz - \omega t) + \hat{\jmath}\, E_{0y}\, cos(kz - \omega t + \varphi)$$

where $\varphi$ is a phase shift between the $\hat{\imath}$ and $\hat{\jmath}$ directions. If $\varphi = 0$, the electric field oscillates along a line. This is linearly polarized light. The angle of the polarization is determined by the relative magnitudes of $E_{0x}$ and $E_{0y}$. If $\varphi = \pm \pi/2$, and $E_{0x} = E_{0y}$ the electric field vector traces out a circle in the x−y plane. This describes *circularly polarized light*. If $\varphi \neq 0, \pm \pi/2$ and/or $E_{0x} \neq E_{0y}$, the light is elliptically polarized. The electric field traces out an ellipse, that in general can have any major to minor axis ratio and an arbitrary angle.

Polarizers are devices that can set or measure the polarization state of a plane wave.

Malus's Law states that light having intensity I(0) polarized at an angle $\theta$ with respect to a reference direction and passing through an analyzer at an angle $\phi$ with respect to the reference direction, will have an intensity of

$$I(\phi) = I(0) Cos^2(\theta - \phi)$$

If the beam goes through a second analyzer oriented at an angle $\psi$ with respect to the reference direction, the resulting intensity is given by

$$I(\psi) = I(\phi) Cos^2(\phi - \psi)$$
$$I(\psi) = I(0) Cos^2(\theta - \phi) Cos^2(\phi - \psi)$$

**Reflection of Linearly Polarized Light**

Light incident on a planar interface between two media can be decomposed into two polarizations states:

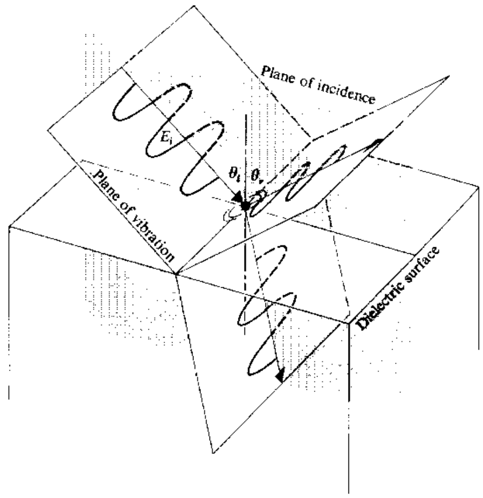
a) Electric field perpendicular to the plane of incidence [⊥]. Reflectance:

$$R_{\perp} = \frac{sin^2(\theta_{inc} - \theta_{trans})}{sin^2(\theta_{inc} + \theta_{trans})}$$

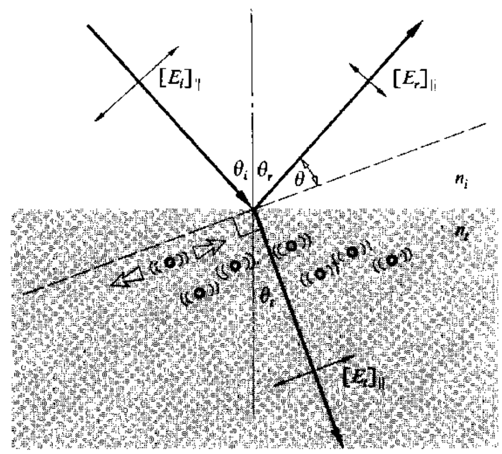
b) Electric field parallel to the plane of incidence (or scattering plane) [||]. Reflectance:

$$R_{\parallel} = \frac{tan^2(\theta_{inc} - \theta_{trans})}{tan^2(\theta_{inc} + \theta_{trans})}$$

Note that the transmitted angle is not usually measured in a reflection experiment. It can be found from Snell's law: $n_{inc}\, sin\theta_{inc} = n_{trans}\, sin\theta_{trans}$.




**Perpendicular case (⊥)**



**Parallel Case (||)**

The case of electric field in the plane of incidence (||) is of interest here. The refracted ray induces oscillations of the electrons in the glass in a direction perpendicular to the refracted ray direction (see above figure, right side). These electrons radiate like dipoles. Consider the special case when the reflected ray is exactly aligned with the axis of oscillation of the electric dipoles. There is a node in the radiation pattern of a dipole antenna in this direction, and the reflected ray will have zero intensity. The condition for this extinction of the reflected ray is

$$\theta_{reflected} + \theta_{transmitted} = 90°$$

or using Snell's Law

$$n_{inc} \sin\theta_{Brewster} = n_{trans} \sin\theta_{transmitted}$$

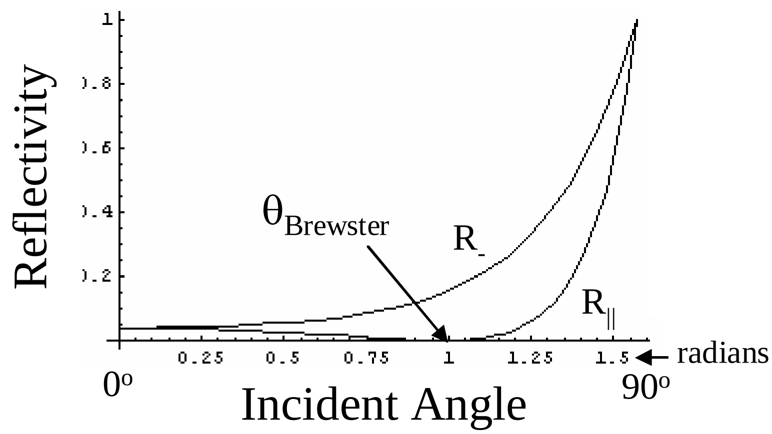
where $n_{inc}$ and $n_{trans}$ are the indices of refraction in the incident and transmitted regions, and $\theta_{Brewster}$ is the special case incident angle. This simplifies to

$$\theta_{Brewster} = \tan^{-1}\left(\frac{n_{trans}}{n_{inc}}\right)$$

Note (from the equation) that the reflectance for parallel polarization is zero at the Brewster angle because $\tan^2(\theta_{Brewster} + \theta_t) = \tan^2(90^o) = \infty$.

**Reflectivity vs. Angle**

Using the Fresnel equations from $P^3$ Chap. 23, you can calculate the reflection coefficient for light polarized in the directions parallel ($R_{||}$) and perpendicular ($R_\perp$) to the scattering plane as a function of the angle of incidence. An example plot for air and a certain type of glass is shown below:




Note that for parallel polarization, most of the light at small angles of incidence is poorly reflected. This means that most of the reflected light is polarized in a direction perpendicular to the plane of incidence. One can use this fact to make sunglasses that reduce glare by using a polarizing filter to cut out the light of the remaining polarization (aka Polaroid sunglasses).

**"Unpolarized" Light**

If light is monochromatic, it is polarized. It must be some form of elliptically polarized light (where we consider linear and circular to be limiting cases.) For light to be "unpolarized" requires a random phase variation between the x and y components of the field, and some time averaging. All light is always fully polarized at any instant in time. Unpolarized light is only possible if there are multiple frequencies present.

**Degree of Polarization**

The degree of polarization is basically a measure of the extent to which the light we are studying is polarized and is given by:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}},$$

where $I_{min}$ and $I_{max}$ are the minimum and maximum intensities of the light, respectively, as an analyzer is rotated. For partially polarized light, this can also be written as:

$$V = \frac{I_p}{I_p + I_{up}},$$

where $I_p$ is the intensity of the polarized portion and $I_{up}$ is the intensity of the unpolarized portion. Sometimes $V$ is called the fringe visibility.