## Polarization Review

Phys 375

## Polarization

The polarization state of a plane wave is specified by the direction of the electric field vector. The polarization state can be linear, elliptical, or circular.

Polarizers are devices that can set or measure the polarization state of a plane wave.

Malus's Law states that light having intensity $\mathrm{I}(0)$ polarized at an angle $\theta$ with respect to a reference direction and passing through an analyzer at an angle $\phi$ with respect to the reference direction, will have an intensity of

$$
I(\phi)=I(0) \operatorname{Cos}^{2}(\theta-\phi)
$$

If the beam goes through a second analyzer oriented at an angle $\psi$ with respect to the reference direction, the resulting intensity is given by

$$
\begin{aligned}
& I(\psi)=I(\phi) \operatorname{Cos}^{2}(\phi-\psi) \\
& I(\psi)=I(0) \operatorname{Cos}^{2}(\theta-\phi) \operatorname{Cos}^{2}(\phi-\psi)
\end{aligned}
$$

## Reflection of Linearly Polarized Light

Light incident on a planar interface between two media can be decomposed into two polarizations states:
a) Electric field perpendicular to the plane of incidence $[\perp]$.

$$
\text { Reflectance: } R_{\downarrow}=\frac{\sin ^{2}\left(\theta_{i}-\theta_{t}\right)}{\sin ^{2}\left(\theta_{i}+\theta_{t}\right)}
$$

b) Electric field parallel to the plane of incidence (or scattering plane) [\|].

$$
\text { Reflectance: } R_{\|}=\frac{\tan ^{2}\left(\theta_{i}-\theta_{t}\right)}{\tan ^{2}\left(\theta_{i}+\theta_{t}\right)}
$$

Note that the transmitted angle is not usually measured in a reflection experiment. It can be found from Snell's law: $n_{i} \sin \theta_{i}=n_{t} \sin \theta_{t}$.


Perpendicular case ( $\perp$ )


Parallel Case (||)

The case of electric field in the plane of incidence $(\|)$ is of interest here. The refracted ray induces oscillations of the electrons in the glass in a direction perpendicular to the refracted ray direction (see above figure, right side). These electrons radiate like dipoles. Consider the special case when the reflected ray is exactly aligned with the axis of oscillation of the electric dipoles. There is a node in the radiation pattern of a dipole antenna in this direction, and the reflected ray will have zero intensity. The condition for this extinction of the reflected ray is

$$
\theta_{\text {reflected }}+\theta_{\text {transmitted }}=90^{\circ}
$$

or using Snell's Law

$$
n_{\text {inc }} \sin \theta_{\text {Brewster }}=n_{\text {trans }} \sin \theta_{\text {transmitted }}
$$

where $n_{\text {inc }}$ and $n_{\text {trans }}$ are the indices of refraction in the incident and transmitted regions, and $\theta_{\text {Brewster }}$ is the special case incident angle. This simplifies to

$$
\theta_{\text {Brewsster }}=\tan ^{-1}\left(\frac{n_{\text {trans }}}{n_{\text {inc }}}\right)
$$

Note (from the equation) that the reflectance for parallel polarization is zero at the Brewster angle because $\tan ^{2}\left(\theta_{\text {Brenster }}+\theta_{t}\right)=\tan ^{2}\left(90^{\circ}\right)=\infty$.

## Reflectivity vs. Angle

Using the Fresnel equations from $\mathrm{P}^{3}$ Chap. 23, you can calculate the reflection coefficient for light polarized in the directions parallel $\left(\mathrm{R}_{\|}\right)$and perpendicular $(\mathrm{R} \perp)$ to the scattering plane as a function of the angle of incidence. An example plot for air and a certain type of glass is shown below:


Note that for parallel polarization, most of the light at small angles of incidence is poorly reflected. This means that most of the reflected light is polarized in a direction perpendicular to the plane of incidence. One can use this fact to make sunglasses that reduce glare by using a polarizing filter to cut out the light of the remaining polarization (aka Polaroid sunglasses).

