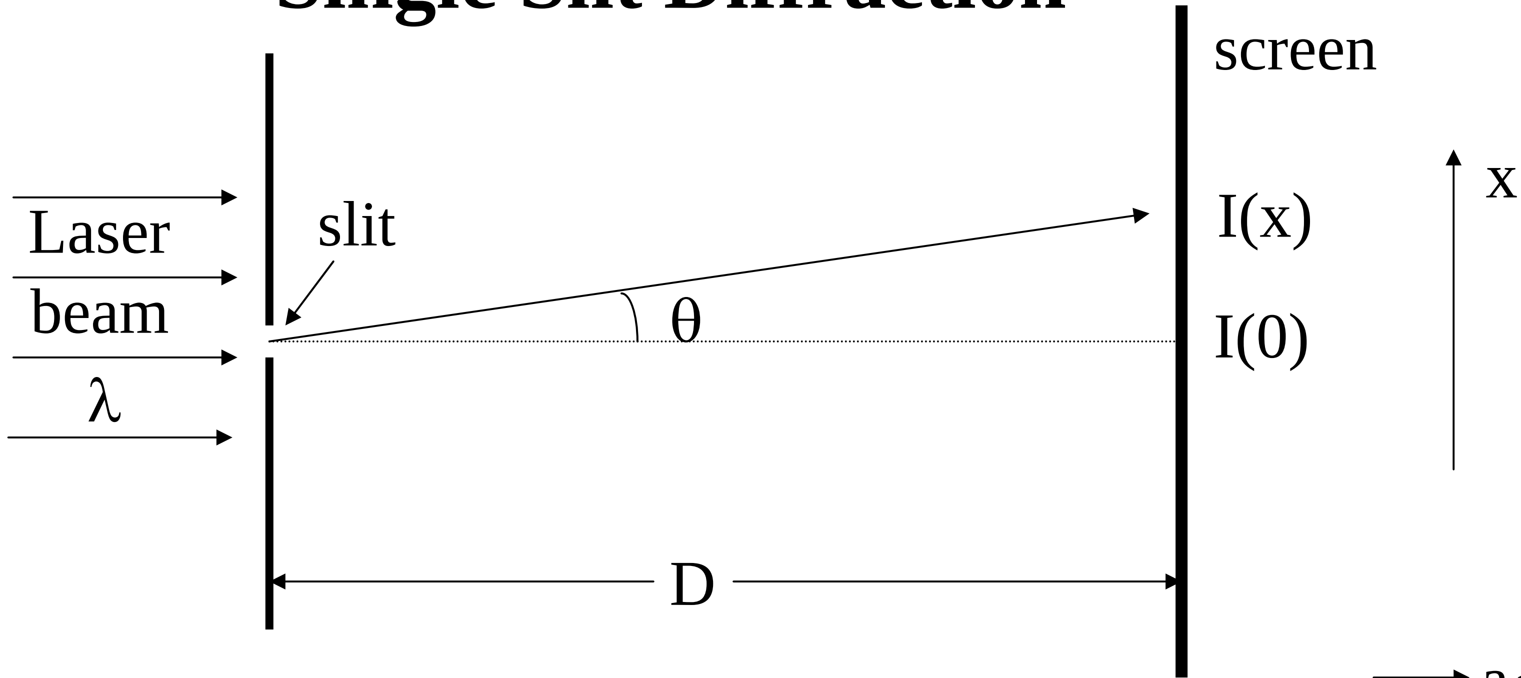
Single Slit Diffraction



Intensity distribution on the screen:

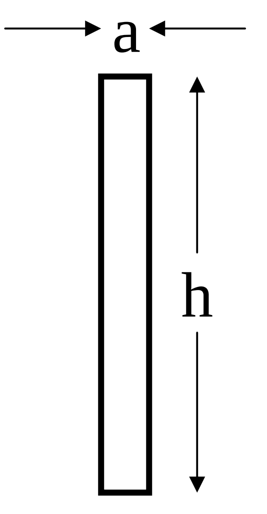
$$I(\theta) = I(0)\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{Eq. (1)} \qquad \alpha = \left(\frac{\pi a}{\lambda}\right)\sin\theta$$

The minima of this diffraction pattern occur at:

$$\alpha_{\min} = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$$

If D >> a, we can approximate α as:

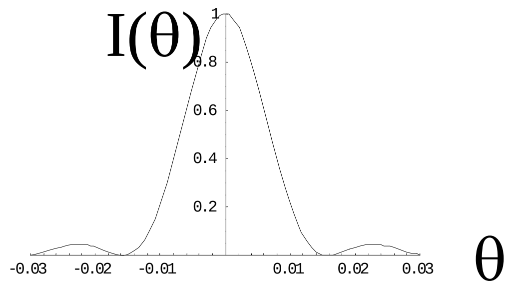
$$\alpha \cong \frac{\pi a}{\lambda}\frac{x}{D}$$



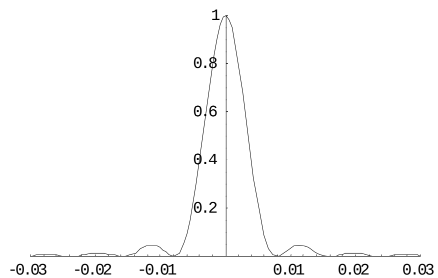
slit geometry

Single Slit Diffraction

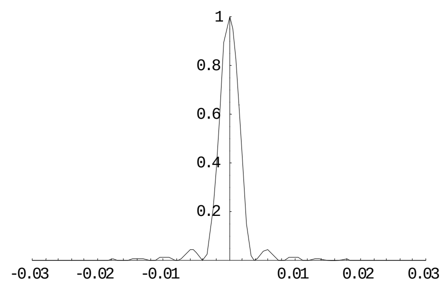
$$I(x) = I(0)\left(\frac{\sin(\pi a x/\lambda D)}{(\pi a x/\lambda D)}\right)^2$$



a = 40 μm



a = 80 μm



a = 160 μm

The diffraction angle θ_{diff} is defined as the half-width of the main diffraction peak:

$$\alpha_{\min} = \pi, \quad \text{so} \quad \pi = \left(\frac{\pi a}{\lambda}\right)\sin\theta_{diff}$$

and

$$\theta_{diff} \cong \frac{\lambda}{a}$$

$\lambda = 0.6328\ \mu\text{m}$

N Slit Diffraction

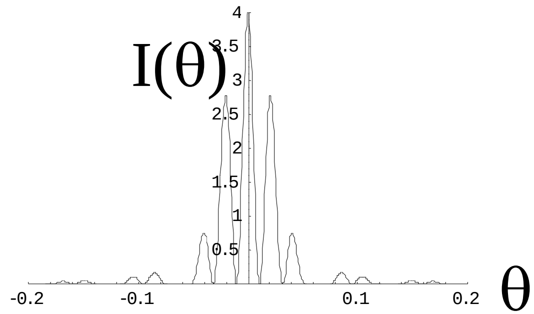
$$I(x) = N^2 I(0)\left(\frac{\sin(\pi a x/\lambda D)}{(\pi a x/\lambda D)}\right)^2\left(\frac{\sin(N\pi d x/\lambda D)}{N\sin(\pi d x/\lambda D)}\right)^2$$

Note that:

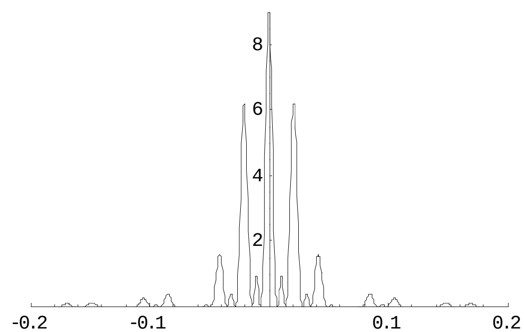
$$\underset{x\to m\pi}{L}\ \frac{\sin(Nx)}{\sin(x)} = \pm N$$

where $m = 0, \pm 1, \pm 2, \ldots$

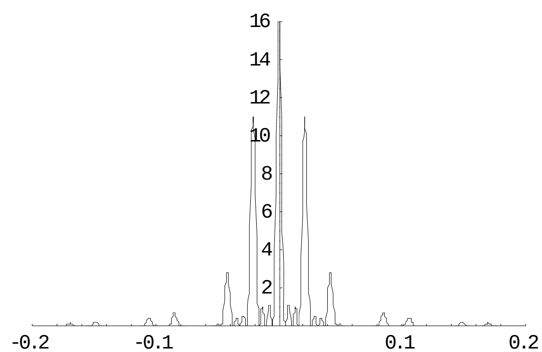
Hence $I(x=0) \sim N^2$



N = 2



N = 3



N = 4

λ = 0.6328 μm
a = 10 μm
d = 30 μm

Fourier Transform of Sinc[γx]^2

$$f(x) = \left(\frac{Sin[\gamma\, x]}{\gamma\, x}\right)^2 \quad \text{with} \quad \gamma = \frac{\pi\, a}{D\lambda}, \qquad \alpha \cong \frac{\pi\, a}{D\lambda} x$$

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FourierTransform[Sin[γ x]^2/(γ x)^2, x, ω]
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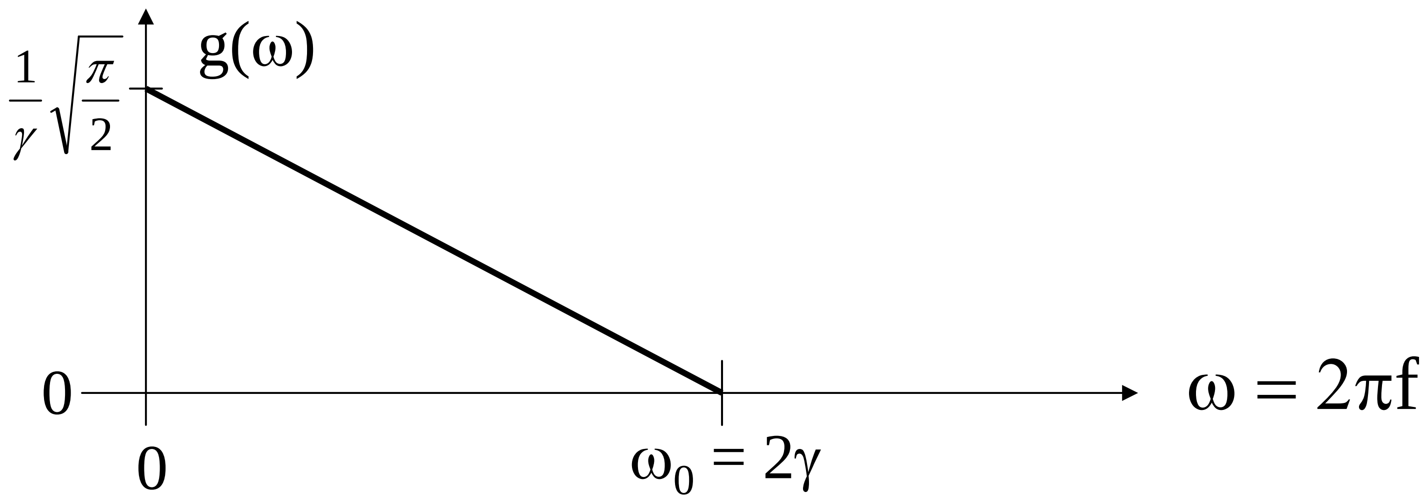
$$g(\omega) = FT[f(x)] = \frac{\sqrt{\frac{\pi}{2}}\,\left((2\gamma-\omega)\,\text{Sign}[2\gamma-\omega] - 2\omega\,\text{Sign}[\omega] + (2\gamma+\omega)\,\text{Sign}[2\gamma+\omega]\right)}{4\gamma^2}$$

Mathematica



We can relate ω_0 to the diffraction angle because $\alpha_{min} = \pi$ there, yielding $x_{min} = \lambda D/a$, and finally $1/f_0 = x_{min}$, where $\omega_0 = 2\pi f_0$. Hence the linear frequency at which the FT goes to zero is exactly equal to the inverse of the distance to the first minimum in the single slit diffraction pattern! This result can be used to estimate the slit width .