Department of Physics
University of Maryland, College Park

Assignment 7, Physics 374 — Due Tuesday, April 13, 2010

Note: In solving math problems, you have to provide the details of intermediate steps. Without those steps, you cannot get full credit.

Problem 1

Consider real functions $f(x)$ defined on the interval $(-L, L)$. The scalar product of functions $f(x)$ and $g(x)$ is defined as

$$(f \cdot g) = \int_{-L}^{L} f(x)g(x)dx \tag{1}$$

Consider function $u_n(x) = A\cos(n\pi x/L)$, for what choice of A, $u_n(x)$ has the norm equal to 1? i.e. $(u_n \cdot u_n) = 1$. Show that

$$(u_n \cdot u_m) = \delta_{nm} \tag{2}$$

Problem 2

What is the Fourier expansion of the function $f(x) = -x^2 + 1$ defined in the interval $(-1, +1)$ and periodic outside of it?

Problem 3

The ground state of the hydrogen atom has the following wave function

$$\psi(\vec{r}) = \lambda e^{-r/a} \tag{3}$$

work out its 3-dimensional Fourier transformation by going to spherical coordinates.

Problem 4

Using complex integration and contour integral, work out the Fourier transformation of

$$\frac{1}{x^2 + a^2} \tag{4}$$

Problem 5

Show that the function $\exp(z)$ is analytic at $z = 0$. Show that the function $\sqrt{z}$ is not analytic at $z = 0$.

Problem 6

If the Fourier transformation of $f(t)$ is $F(\omega)$, and that of $h(t)$ is $H(\omega)$, show that

$$\int_{-\infty}^{\infty} F(\omega)H^*(\omega)e^{-i\omega t} = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t+\tau)h^*(\tau)d\tau \tag{5}$$

Problem 7

Prove that

$$\int_0^\infty \frac{\sin^2 x}{x^2} dx = \pi/2 \tag{6}$$

Problem 8

The one-dimensional neutron diffusion equation with a source is

$$-D\frac{d^2\phi(x)}{dx^2} + K^2\phi(x) = Q\delta(x) \tag{7}$$

where$D$, K and Q are constants. Apply a Fourier transform to solve the equation in Fourier space.