Department of Physics
University of Maryland, College Park

Assignment 6, Physics 374 — Due Tuesday, April 6, 2010

Note: In solving math problems, you have to provide the details of intermediate steps. Without those steps, you cannot get full credit.

Problem 1

Consider a cylindrical symmetric flow in which the velocity field is independent of z, but a function of $r = \sqrt{x^2 + y^2}$,

$$\vec{v} = \hat{\phi} v(r) \tag{1}$$

where $\hat{\phi}$ is a unit vector for the polar angle and $v(r)$ is some function of r. Find the curl of the velocity field.

Problem 2

a). Following problem 1). What velocity field gives a constant curl.

b). Find the curl for velocity field $v_x = v_0 \exp(-y^2/L^2)$, $v_y = v_z = 0$.

Problem 3

See page 98 in the textbook. Verifying the Stokes theorem (9.2) by solving problem a) in that page.

Problem 4

Consider an infinitely-long thin wire along the z-direction at $x = y = 0$. Suppose the total electric current following in the wire is I. Use $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$, where $\vec{B}$ is the magnetic field and $\vec{j}$ is the current density, calculate the magnetic field generated by the current using Stokes theorem. Once you have the magnetic field, calculate its curl in the infinite thin-wire limit at every point in space, including $x = y = 0$. [Hint: you shall have a 2D delta function.]

Problem 5 Compute the Taylor expansion of $1/(1-x)$ at $x = 1/2$. What is its radius of convergence? why?

Problem 6 Calculate the Gaussian integrals

$$\int_0^\infty x^n e^{-\lambda x^2}, \tag{2}$$

for $n = 1, 2, 3, ...$, from the basic Gaussian integral $n = 0$ by taking derivatives with respect to λ. Using the result, calculate the integral $\int_0^\infty e^{-\lambda x^2 - \alpha x^4} dx$ by developing a Taylor series in α. Is it convergent series?

Problem 7 Show that $\epsilon/(x^2 + \epsilon^2)$ as $\epsilon \to 0$ is proportional to a δ function. Write done explicitly what it is in term of a delta function.

Problem 8 Calculate the integral

$$\int_1^\infty \delta(\sin x) e^{-x} dx \tag{3}$$

where the argument of a delta function is a sine-function.