Department of Physics
University of Maryland, College Park

Assignment 5, Physics 374 — Due Tuesday, March 30, 2010

Note: In solving math problems, you have to provide the details of intermediate steps. Without those steps, you cannot get full credit.

Problem 1

Consider a scalar field $V(r) = 1/r^3$, calcualte its gradient ∇V in both rectangular and spherical coordinate systems.

Problem 2

Consider a path integral of the above ∇V from $\vec{r}_1 = (1, 1, 0)$ to $\vec{r}_2 = (2, 2, 0)$. Choose two different paths (**1**) from $(1, 1, 0)$ to $(2, 1, 0)$ along a straight line and then to $(2, 2, 0)$ along another straight line, and similarly (**2**) from $(1, 1, 0)$ to $(1, 2, 0)$ and to $(2, 2, 0)$, and show that the results are the same. Can you get the result without doing the integration?

Problem 3 Read section 5.6 and work through problems c), d), e) and f) so that you get an expression for gradient in spherical coordinates.

Problem 4

The magnetic field of a magnetic dipole oriented in the z-direction has the following form,

$$\vec{B} = \frac{3m\vec{r}z}{r^5} - \frac{m\vec{k}}{r^3} \tag{1}$$

Use the expression for divergence in spherical coordinates (6.32) to calculate $\vec{\nabla} \cdot \vec{B}$.

Problem 5

Compute the flux of the vector field $\vec{V} = (x + y + z)\vec{k}$ through a sphere of radius R centered at the origin by explicitly computing the integral that defines the flux. Solve the same problem, using Gauss's law.

Problem 6

Gravitational field $\vec{g}(\vec{r})$ satisfies the following equation

$$\vec{\nabla} \cdot \vec{g} = -4\pi G\rho(\vec{r}) \tag{2}$$

where $\rho(\vec{r})$ is the mass density and G is the gravitational constant. For a spherically symmetric mass density, ρ is a function of r only and the gravitational field has the form

$$\vec{g}(\vec{r}) = g(r)\hat{r} \tag{3}$$

Use Gauss's law to show that

$$\vec{g} = -\frac{GM}{r^2}\hat{r} \tag{4}$$

where M is the mass enclosed in a sphere of radius r.