Department of Physics
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Assignment 4, Physics 374 — Due Tuesday, March 9, 2010

Note: In solving math problems, you have to provide the details of intermediate steps. Without those steps, you cannot get full credit.

Problem 1 The cyclic group S_n is made of elements e, a, a^2, ..., a^{n-1}, where e is the identity element and a satisfies $a^n = e$. Work out the group multiplication table for $n = 3$.

Problem 2

Consider geometrical shapes in 2D (such as a triangle, square, pentagon, etc). Which shape has the symmetry represented by S_3 (see problem 1)? What is the operation corresponding to the identity element? What operation corresponding to a?

Problem 3 Show all integers under ordinary addition form a group. [Hint: identify "multiplication operation" and check the four conditions to form a group.]

Problem 4

A subgroup H of group G has elements h_i. Let x be a fixed element of the original group G and not a member of H. The transformation xh_ix^{-1}, $i = 1, 2..$ generates a conjugate subgroup xHx^{-1}. Show that the conjugate subgroup satisfies each of the four group postulates and therefore is a group.

Problem 5

Consider rotations in x-y plane around z-axis. The most general rotation matrix

$$R(\phi) = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

forms a group SO(2) under matrix multiplication. a) find identity element, b) find R^{-1}, c), for small ϕ, we can write

$$R(\phi) = 1 + \phi J + \mathcal{O}(\phi^2) + ... \tag{2}$$

identify matrix J (generator). c) Show that for finite ϕ, $R(\phi) = e^{\phi J}$. [Hint: use the Taylor series for exponential function.]

Problem 6

SU(2) group contains all unitary complex matrices ($U^\dagger U = 1$, where $\dagger$ means hermitian conjugation: complex conjugation + transpose) in 2D with $\det U = 1$. Show that U has three independent parameters.