Department of Physics
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Assignment 3, Physics 374 — Due Tuesday, March 2, 2010

Note: In solving math problems, you have to provide the details of intermediate steps. Without those steps, you cannot get full credit.

Problem 1

Director product of two vectors $\vec{A} = \vec{i} + 3\vec{j} + 2\vec{k}$ and $\vec{B} = 2\vec{i} + 4\vec{j} + \vec{k}$ yields second-order tensor C_{ij}, which can be written as a square matrix. However, the result depends on the order of the product, namely either $C_{ij} = A_i B_j \equiv B_j A_i \equiv (\vec{A}\vec{B}^T)_{ij}$ (here $\equiv$ means *identical*) or $C'_{ij} = A_j B_i \equiv B_i A_j \equiv (\vec{B}\vec{A}^T)_{ij}$. Work out the matrices C and C' using matrix product (a vector can be seen as a column vector), and find out what is the relationship between them.

Problem 2

Construct a projection operator (matrix) such that it will project any 3-dimensional vector on to the direction of $(1, 1, 1)$. Find the eigenvalues and eigenvectors of the above projection operator. Hint: 1) when you have two eigenvalues that are the same, we say that the eigenvalue has two-fold degeneracy. In this case, there is a considerable freedom in choosing the eigenvectors. 2) If you can provide a good argument for what the result shall be, you don't actually need to make a detailed calculation.

Problem 3

Starting with a basis $\{\vec{e}_i\}$, expand an arbitrary vector $\vec{A}$ in this basis $\sum_i A_i \vec{e}_i$. Work out A_i as the scalar product of $\vec{e}_i$ and $\vec{A}$ and substitute this into the above expansion. Since the expansion works for any $\vec{A}$, you get a condition which must be true for the basis $\{\vec{e}_i\}$. Work out this condition (the closure relation).

Problem 4

If a matrix has three eigenvectors $\vec{v}_1 = \frac{1}{\sqrt{3}}(1, 1, 1)$, $\vec{v}_2 = \frac{1}{\sqrt{2}}(1, 0, -1)$, and $\vec{v}_3 = \frac{1}{\sqrt{6}}(1, -2, 1)$ with eigenvalues, $1, 2, 3$, respectively, construct this matrix..

Problem 5

In the spherical system of coordinates, $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, and $z = r\cos\theta$. The basis for the spherical coordinates can be defined as the derivatives of the vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ with respect to individual coordinates, i.e.,

$$\hat{r} = \frac{\partial \vec{r}}{\partial r}, \quad \hat{\theta} = \frac{\partial \vec{r}}{r\partial\theta}, \quad \hat{\phi} = \frac{\partial \vec{r}}{r\sin\theta\partial\phi} \tag{1}$$

Using this, work out in detail three unit vectors $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$. Note that the basis is now *local* in the sense that it depends on the spatial location, whereas in the rectangular system, it is the same everywhere.

Problem 6

Inverting the relations in Problem 5 to obtain $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$ in terms of $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$. This can be done most efficiently by recognizing that the transformation matrix is orthogonal.