Department of Physics
University of Maryland, College Park

Assignment 2, Physics 374 — Due Tuesday, Feb. 23, 2010

Problem 1

If a new system of coordinates is obtained by rotating around the x-axis by 30 degree in a positive sense (use the right-handed rule), what is the transformation matrix R for the basis vectors $\vec{e}_i$? Verify that this matrix is an orthogonal one. Show that the determinant of this matrix is 1. Is this true for all orthogonal matrices?

Problem 2

In a basis $\{\vec{e}_i\}$, an operator is a 3×3 symmetric matrix

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix} \tag{1}$$

Find its eigenvalues and the corresponding normalized eigenvectors. Show that these eigenvectors are orthogonal to each other. [You must show the details of the calculation to get full credit.]

Problem 3

Following Problem 2. If we use these eigenvectors as a new basis $\{\vec{e}'_i\}$, what is the transformation matrix R to the new basis? Is this transformation matrix orthogonal? Verify your answer.

Problem 4

Following Problems 2 and 3. Suppose we have an operator B, which has the expression in the original basis as a

$$B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{2}$$

Calculate the matrix B in the new basis using the second-order tensor transformation formula. Verify the result for the $\{11\}$ element, defined as $B'_{11} = \vec{e}'_1 \cdot (B\vec{e}'_1)$, by working out explicitly the basis transformation calculation.

Problem 5

Consider a fourth order tensor T_{ijkl} made of the direct product of a vector $v = (1, 1, 2)$ and the above matrix B, $T_{ijkl} = v_i v_j B_{kl}$. What is T_{1121} is this basis? What is T_{1121} in the new basis define in Problem 3?

Problem 6

Can you write the determinant of a proper orthogonal matrix (determinant=1) in terms of the ϵ^{ijk} tensor? Can you show that this implies that ϵ^{ijk} is an invariant tensor? Can you generalize the conclusion to an n-dimensional tensor $\epsilon_{i_1,i_2,\ldots,i_n}$.