Instructions:

Do not open this examination until the proctor tells you to begin.

1. When the proctor tells you to begin, **write your full name and section number at the top of every page.** This is essential since this exam booklet will be separated for grading.

2. Do your work for each problem on the page for that problem. You might find it convenient to either do your scratch work on the back of the page before starting to write out your answer or to continue your answer on the back. **If part of your answer is on the back, be sure to check the box on the bottom of the page so the grader knows to look on the back!**

3. On all the problems your answers will be evaluated at least in part on how you got them. If explanations are requested, more than half the credit of the problem will be given for the explanation. LITTLE OR NO CREDIT MAY BE EARNED FOR ANSWERS THAT DO NOT SHOW HOW YOU GOT THEM. Partial credit will be granted for correct steps shown, even if the final answer is wrong.

4. Write clearly and logically so we can understand what you are doing and can give you as much partial credit as you deserve. We cannot give credit for what you are thinking — only for what you show on your paper.

5. All estimations should be done to the appropriate number of significant figures.

6. At the end of the exam, write and sign the honor pledge in the space below: “I pledge on my honor that I have not given or received any unauthorized assistance on this examination.”

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***Good Luck***
1. (30 points) The Euler method treats Newton’s second law (N2) as a set of “marching orders.” Given the “state of an object (its position and velocity) at a particular time, \( t \), it uses N2 to calculate (predict) the state of the object (its position and velocity) at a later time, \( t + \Delta t \).

(a) For a cart on a spring, taking the coordinate to start at 0 at the equilibrium position, write N2 for the cart. (5 pts)

(b) Rewrite N2 for the time interval from \( t \) to \( t + \Delta t \) and show how to rewrite this form to give Euler’s method — a way of calculating the position and velocity at a time \( t + \Delta t \) given the position and velocity at a time \( t \). (10 pts)

(c) If at \( t = 0 \), the position of the object is \( x(0) = A \) and the velocity is \( v(0) = 0 \), find the position and velocity of the mass as given by the first two steps of the Euler method. (“Two” does not include the initial state.) How good are these answers? (15 pts)
2. (20 points) Suppose you have a very long line of charge with uniform charge per unit mass, \( \lambda \), lying along the y-axis. (Only part of it is shown in the figure.) Using Gauss's Law, it is fairly easy to show (Don't do it!) that the electric field vector (i.e. the electric force per unit charge) in the x-y plane due to this line charge is given by \( \frac{\lambda}{2\pi \varepsilon_0 x} \hat{i} \).

There is also a uniform gravitational field (i.e. the gravitational force per unit mass) given by \(-g \hat{j}\).

If you release an object with a charge \( q \) and mass \( m \) from rest at \( x = 10 \text{ cm}, y = 40 \text{ cm} \) it will follow the curved path shown by the solid line. The parameters of the problem are such that the object passes through the point \( x = 50 \text{ cm}, y = 10 \text{ cm} \).

(a) Starting with the fundamental definition of work, find the work done by the combined electric and gravitational forces on the object if it moves not along the curved path but along the path shown by the dotted lines. (10 pts)

(b) Is the result you obtained in the previous section the same or different from the work that is done by the combined electric and gravitational forces along the true (curved) path, shown by a solid line? Explain. (5 pts)

(c) How fast is the object going when it reaches the point \( x = 50 \text{ cm}, y = 10 \text{ cm} \)? (Assume the object moved along the solid line path.) (5 pts)
3. (15 points) When flying to Savannah this week on a small plane, I looked out the window and noticed that a typical automobile almost directly below appeared to be about the size of a dime held at arm’s length. Estimate how high the plane was above the ground. Be sure to clearly state your assumptions and how you came to the numbers you estimated, since grading on this problem will be mostly based on your reasoning, not on your answer.
4. (10 points) In the first third of this class we talked about two different kinds of vectors associated with locations in space. What distinguished these vectors was how they changed when we change the way we choose to describe them. Explain what the two kinds of vectors are and give a physical example of each kind.
5. (25 points) Consider the simple case of a ball thrown straight up starting at a position \( y = 0 \) with an initial upward velocity of magnitude \( v_y(t=0) = v_0 \). Treat gravity in the flat-earth approximation and ignore air resistance.

(a) Using dimensional analysis, create a “natural length” from the parameters of this problem. (What are appropriate constants to use for this task?) (5 pts)

(b) Find the position, \( y(t) \), and velocity, \( v_y(t) \), of the ball as a function of time. (5 pts)
   *Hint:* One good way to do this is to start from the equation of motion, N2, and integrate.

(c) Find the time, \( t_m \), at which the ball reaches its maximum height. (5 pts)

(d) Make a Taylor series for the height of the ball, \( y(t) \), about \( t_m \). Keep terms through the second order. (10 pts)