Physics 374 Homework 9

1) In class we showed a general argument why the lowest nonvanishing multipole coefficient was independent of the origin. We also showed explicitly for the case where the m=0 multipole was absent that the m=1 multipole coefficient was independent of the origin. The explicit calculation was done by shifting the origin in the integral of the m=1 moment. In this problem I would like you to use the same method to show explicitly for the case where the m=0 and m=1 multipole coefficients vanish the m=2 multipole coefficient is indpendent of the origin.

2) Consider a cylindrical surface of radius R. Suppose the potential on the surface of the cylinder is fixed to the following value: $\Phi(R,\vartheta)=V_0\left(\sin^2(\vartheta)-\frac{1}{2}\right)$ and that there are no charges away from the surface of the cylinder.
   a) Find the value of $\Phi(r,\vartheta)$ for r<R (Hint only one multipole will contribute)
   b) Find the electric field for r<R.
   c) Find the value of $\Phi(r,\vartheta)$ for r>R

3) Consider two cylindrical surface of radii R_1 and R_2. Suppose the potential on the surface of the two cylinders are fixed to the following values: $\Phi(R_1,\vartheta)=V_1\left(\sin^2(\vartheta)-\frac{1}{2}\right)$ and $\Phi(R_2,\vartheta)=V_1\left(\cos^2(\vartheta)-\frac{1}{2}\right)$; there are no charges away from the surfaces of the cylindesr.
   Find the value of $\Phi(r,\vartheta)$ for $R_1<r<R_2$ (Hint only one multipole will contribute)

4) Consider the following set up. There is vacuum in the region $|x^2-y^2|\le R^2$ where R is a fixed parameter. The region $|x^2-y^2|>R^2$ is filled with conducting material which can be held fixed potentials Note that there are four distinct conducting regions---in polar coordinates they are: i) $-\frac{\pi}{4}<\vartheta<\frac{\pi}{4}$; ii) $\frac{\pi}{4}<\vartheta<\frac{3\pi}{4}$; iii) $\frac{3\pi}{4}<\vartheta<\frac{5\pi}{4}$; and iv) $\frac{5\pi}{4}<\vartheta<\frac{7\pi}{4}$. Suppose that regions i) and iii) are held at a potential of $+V_0$ while regions ii) and iv) are held at $-V_0$. Find the potential for the region $|x^2-y^2|\le R^2$. This involves solving the Laplace equation subject to a boundary condition. Hint: You should be able to convince yourself that only the m=± 2 terms will contribute.