PHYS 374 Homework 4

1. Consider a particle of mass *m* moving in the following potential:
   $U(x) = \dfrac{U_0}{1+(x/L)^2}$ with $U_0 > 0$. The particle is released from rest very near the top with x>0.

a) Derive an approximate expression for the time it takes for the particle to move from this point down to x/L=1/2.

b) What is the condition for "very near the top"

2. Consider the system described in problem 1.

a) Derive an expression for the time to move from an initial point near the top at rest down to x/L=1/2 using the energy conservation method described in class. The expression can be left in the form of an integral

b) For an initial x/L=.0001 find the period using the exact expression in part a), and the approximate expression from problem 1. Is the approximation as good as you expected. Explain.

3. Verify explicitly that the Lorentz transformation:
   $$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$ with $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$ keeps the Lorentz interval invariant, *i.e.* $t'^2 - x'^2 - y'^2 - z'^2 = t^2 - x^2 - y^2 - z^2$

4. By making two successive Lorentz boosts in the x direction, one of velocity v_1 and the other of velocity v_2, derive the relativistic velocity addition formula $v_{total} = \dfrac{v_1 + v_2}{1 + v_1 v_2}$.