Physics 374 Homework 10

1. Consider the situation discussed in class in which an approximately elliptical conductor (whose shape is given by $R_1(\phi) = \sqrt{\alpha^2 \cos(\phi) + \beta^2 \sin(\phi)}$) is held at potential *V* while surrounding the ellipse is a grounded circle of radius *R.* (You may think of this as 2-dimensional problem or as a 3-dimensional problem independent of z). Supposing that the *R=10* α and $\beta = 1.2\ \alpha$ find an approximate expression for the potential between the plates by writing the multipole expansion an d matching with fixed points on the surface. You may work up to a maximum m of 2 which means 2 points on the surface. You may choose these two points to be at $\phi=0$ and $\phi=\pi/2$ on each surface.

   a) What are the multipole coefficients?
   b) Make a contour plot of the potential.
   c) Find the electric field expressed in Cartesian coordinates.

2. Consider the same problem in 1.

   a) This time choose the points at $\phi=\pi/6$ and $\phi=\pi/3$ on each surface. What are the multipole coefficients? How do they compare to the coefficients in 1.? Is this sensible?
   b) This time work up to a maximum m of 4 picking your points at $\phi=0$, $\phi= \pi/4$ and $\phi=\pi/2$ on each surface. Are the m=0 and m=2 multipole coefficients essentially unchanged from problem 1? What does this tell us about the convergence of our series.
   c) With out calculating, explain whether picking a maximum m of 4 rather than 2 would be a bigger or smaller effect if instead of the problem 1. we considered the case *R=10*α and $\beta = 1.4\ \alpha$

3. The potential on the surface of a sphere of radius R is fixed at $\Phi(R,\theta) = V_0 \cos^2(\theta)$ with the potential defined to be zero at infinity. There are no charge for r>R or r<R

   a) Use the known Legendre polynomials to express the potential on the surface as the sum of two terms---one with l=0 and the other with l=2, *i.e.* express the potential as
   $\Phi(R,\theta) = c_0\ P_0(\cos(\theta)) + c_2\ P_2(\cos(\theta))$ where c_0 , c_2 are coefficients.
   b) Find the potential and electric field for r>R.
   c) Find the potential for r<R.

4. The potential on the surface of a sphere of radius R_1 is fixed at $\Phi(R_1,\theta)=V_1$ with the potential defined to be zero at infinity. The potential on the surface of a sphere of radius R_2 (with $R_2 > R_1$) is fixed at $\Phi(R_2,\theta)=V_2 \cos(\theta)$. There are no charge for $R_1 < r < R_2$. Find the potential in the region $R_1 < r < R_2$. Hint: Assume only l=0 and l=1 contribute.