

**PHYS 373 (Fall 2021): Mathematical Methods for
Physics II**
Final exam: Monday, December 20, 1.30-3.30 pm.

Read the instructions below and do *not* flip to next page till you are told to do so.

Name:

Student ID:

Useful formulae:

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$ $N_p(x) = \frac{\cos(\pi p)J_p(x) - J_{-p}(x)}{\sin \pi p}$ $\int_0^1 x J_p(\alpha x) J_p(\beta x) dx = 0 \quad (\alpha \neq \beta)$ $c_m = \frac{2m+1}{2} \int_{-1}^1 f(x) P_m(x) dx$ $f(z) = f(x + iy) = u(x, y) + iv(x, y)$ $\frac{d}{dx} \left[x^{-p} J_p(x) \right] = -x^{-p} J_{p+1}(x)$ $\frac{\partial^2}{\partial x^2} T(x, y) + \frac{\partial^2}{\partial y^2} T(x, y) = 0$ $R(z_0) = z \xrightarrow{\lim} z_0 (z - z_0) f(z)$ $\int f(z) dz \text{ (around } C) =$ $f(x) = f(x_0) + \sum_{n=1}^{\infty} \frac{d^n f}{dx^n} \Big _{x=x_0} \frac{(x-x_0)^n}{n!}$ $\int_{-\infty}^{\infty} \frac{P(x)}{Q(x)} e^{imx} dx \text{ (around } C, m > 0) =$ $J_p(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)\Gamma(n+1+p)}$ $ax^2 + bx + c = 0 \rightarrow$	$f(x) = \sum_{l=0}^{\infty} c_l P_l(x)$ $a_n = \frac{1}{l} \int_{-l}^l f(x) \cos \frac{n\pi x}{l} dx$ $b_n = \frac{1}{l} \int_{-l}^l f(x) \sin \frac{n\pi x}{l} dx$ $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ $J_{p-1}(x) - J_{p+1}(x) = 2J'_p(x)$ $N_n(0) = \infty$ $T = \left\{ \begin{matrix} e^{kx} \\ e^{-kx} \end{matrix} \right\} \left\{ \begin{matrix} \sin ky \\ \cos ky \end{matrix} \right\}$ $g(\alpha) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-i\alpha x} dx$ $2\pi i \cdot \text{sum of residues of } f(z)$ $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $2\pi i \cdot \text{sum of residues of } \frac{P}{Q} e^{imz}$ $P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$f(x) = \sum_{-\infty}^{\infty} c_n e^{in\pi x/l}$ $P_3(x) = \frac{1}{2} (5x^3 - 3x)$ $P_1(x) = x$ $P_0(x) = 1$ $J_0(0) = 1$ $J_{n \neq 0}(0) = 0$ <p style="text-align: center;">or $\left\{ \begin{matrix} \sin kx \\ \cos kx \end{matrix} \right\} \left\{ \begin{matrix} e^{ky} \\ e^{-ky} \end{matrix} \right\}$</p> $J_n(\infty) = 0$ $\text{inside } C$ $e^{i\theta} = \cos \theta + i \sin \theta$ $\text{in upper half-plane}$ $\int f(z) dz = 0, \text{ around } C$ $R(z_0) = \frac{g(z_0)}{h'(z_0)}, \text{ if } f = \frac{g}{h}$
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It is necessary to show the details of the derivation and not just the final answer for *all* problems.

This is a closed book exam: crib sheets are not allowed.

In case they are needed, more blank paper and staples are provided.

Please write clearly and if you use the backside of a page, then please indicate so.

Check that there are total of 6 *pages* (containing 6 problems).

Note that some problems have *multiple* parts; so, please read the statement of each problem carefully.

Remember the honor pledge that you signed at the start of the semester.

Problem #	Points scored	Maximum points
1		8
2		14
3		8
4		8
5		12
6		10
Total		60