Modern Physics - HW 1
PHY 371 - Fall 2016

Due Wednesday, September 14 **before** class

I. QUANTIFYING γ FACTORS AND TAYLOR SERIES

Power series expansion (Taylor series) are often used to approximate function when the value of its argument is small. For instance,

$$\frac{1}{(1-x)^n} \approx 1 + \#x + \#x^2 + \mathcal{O}(x^3). \tag{1.1}$$

1) Find the numbers #s.
2) Plot

$$\gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1.2}$$

as a function of v/c.

In the same graph plot the first term of the Taylor approximation ($\gamma \approx 1$), the zeroth plus the second order (proportional to v^2) and the zero plus the second plus the third term (proportional to v^4).

3) Use the results above to find the velocity v for which $\gamma \approx 1.1$.

II. PRACTICE INDEX NOTATION

This may seem random but soon enough we will need it. Let $v = (1, 0, -1)$ and $w = (1, 2, 3)$ and:

$$A_{ij} = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 3 & -1 & 1 \end{pmatrix} \tag{2.1}$$

(we take the convention that the index j in A_{ij} index the different columns and the index i indexes the rows). Compute:

1) $\sum_{i=1}^{3} v_i^2$
2) $\sum_{i=1}^{3} v_i w_i$
3) $\sum_{i,j=1}^{3} v_i w_j$
4) $\sum_{i=1}^{3} v_i^2 w_i$
5)$\sum_{i=1}^{3} A_{ii}$
6)$\sum_{j=1}^{3} A_{ij} v_j$
7)$\sum_{i,j=1}^{3} A_{ij}$
8) $\sum_{i,j=1}^{3} A_{ij} v_i w_j$
9) Which of the combination above are independent of the coordinate system?

III. A RING TO RULE THEM ALL

This is a more evil version of the barn paradox. A ring of diameter D, lying on the horizontal plane falls vertically (negative z direction) with velocity w. At the same time a bar of length $L > D$ (measured at rest), also lying on the horizontal plane, comes from the left moving towards the right (in the positive x direction) with velocity v. Although $L > D$, the Lorentz contracted length $L/\gamma(v) < D$, Alice, who observes this happening in the frame where the fall is vertical, concludes that the bar can go through the ring. Indeed, that's what happens.

However, Bob, an observer moving with the bar is puzzled. For him, the bar has length L while the ring is squished in the x-direction to a diameter $d/\gamma(v)$ so there is no way the bar can go through the ring. Can you explain his faulty reasoning?

This is tricky so I'm going to hold your hand a little bit. Don't get used to it, this the last time.

1) Suppose to events A and B occur at the same time but different x coordinates $x_A < x_B$ as measured in the Alice frame. Do they occur at the same time in the Bob frame? Which one happens first ?

2) What does it mean to I say that the ring is falling vertically lying on the horizontal plane? Can you formulate this condition in terms of some events being simultaneous?

3) Does the ring lie horizontally as measured by Bob?

4) So, what's wrong with Bob's reasoning?

You can be qualitative in this problem. Next week, armed with a more powerful formalism, we will figure out the same puzzle quantitavely.