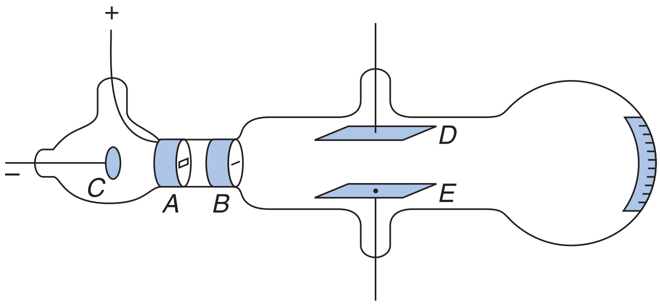
Early Measurements of e/m

Many studies of electrical discharges in gases were done in the late nineteenth century. It was found that the ions responsible for gaseous conduction carried the same charge as did those in Faraday's electrolysis experiments. The year following Zeeman's first calculation of the q/m for atomic particles, J. J. Thomson measured the q/m value for the so-called cathode rays and pointed out that if their charge was Faraday's minimum charge e as determined by Stoney, then their mass was only a small fraction of the mass of a hydrogen atom. He had, in fact, discovered the *electron*. The cathode-ray tube used by Thomson (the apparatus is shown in Figure EM-1) is typical of those used by his contemporaries. It was the forerunner of the television picture tube, the oscilloscope, and a host of video display terminals on everything from word processors and personal computers to video games and radar screens. At sufficiently low pressure, the space near the cathode becomes dark, and as the pressure is lowered still further, this dark space extends across the tube until it finally reaches the glass, which then glows as a result of the energy absorbed from the cathode rays. When apertures are placed at A and B, the glow is limited to a well-defined spot on the glass. This spot can then be deflected by electrostatic or magnetic fields. In 1895, J. Perrin had collected these "cathode rays" on an electrometer and found them to carry a negative electric charge. That direct measurement of the charge-to-mass ratio e/m of electrons by J. J. Thomson in 1897 can be justly considered the beginning of our understanding of atomic structure.

When a uniform magnetic field of strength B is established perpendicular to the direction of motion of charged particles, the particles move in a circular path. The radius R of the path can be obtained from Newton's second law by setting the magnetic force quB equal to the mass m times the centripetal acceleration u^2/R:

$$quB = \frac{mu^2}{R} \quad \Rightarrow \quad R = \frac{mu}{qB} \qquad \textbf{EM-1}$$

Present-day particle physicists routinely use the modern equivalent of Thomson's experiment to measure the momenta of elementary particles. Equation EM-1 is the non-relativistic version of Equation 2-37, that is, with $\gamma = 1$; Thomson, who didn't know about relativity at the time, of course, was fortunate in that the speeds of his "cathode rays" (electrons) were decidedly non-relativistic: that is, the electron speed u were much smaller that the speed of light c, with $u/c \ll 0.2$ (see Figure 2-2). In his first measurement, Thomson determined the velocity from measurements of the total charge and the temperature change occurring when the beam struck an insulated collector. For N particles, the total charge is $Q = Ne$, while the temperature rise is



EM-1 J. J. Thomson's tube for measuring e/m. Electrons from the cathode C pass through the slits at A and B and strike a phosphorescent screen. The beam can be deflected by an electric field between the plates D and E or by a magnetic field (not shown) whose direction is perpendicular to the electric field between D and E. From the deflections measured on a scale on the tube at the screen, e/m can be determined. [*From J. J. Thomson, "Cathode Rays," Philosophical Magazine (5),* ***44,*** 293 (1897).]

proportional to the energy loss $W = N(\frac{1}{2}mu^2)$. Eliminating N and u from these equations, we obtain

$$\frac{e}{m} = \frac{2W}{B^2R^2Q} \qquad \textbf{EM-2}$$

In his second experiment, which came to be known as the *J. J. Thomson experiment*, he adjusted perpendicular B and ξ fields so that the particles were *undeflected* at the screen. This allowed him to determine the speed by equating the magnitudes of the magnetic and electric forces:

$$quB = q\xi \quad \Rightarrow \quad u = \frac{\xi}{B} \qquad \textbf{EM-3}$$

He then turned off the B field and measured the deflection of the particles on the screen. This deflection is made up of two parts (see Figure EM-2). While the particles are between the plates, they undergo a vertical deflection y_1, given by

$$y_1 = \frac{1}{2}at_1^2 = \frac{1}{2}\frac{e\xi}{m}\left(\frac{x_1}{u_x}\right)^2 \qquad \textbf{EM-4}$$

where x_1 is the horizontal distance traveled. After they leave the plates, they undergo additional deflection y_2, given by

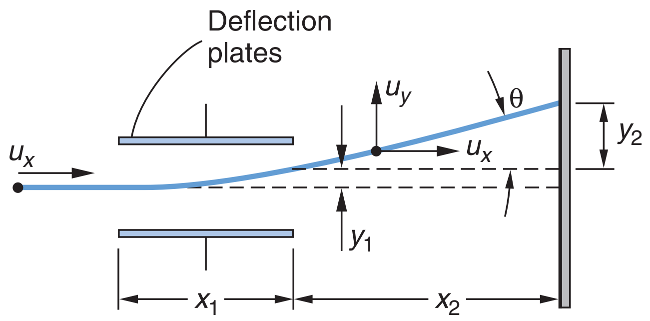
$$y_2 = u_y t_2 = at_1\left(\frac{x_2}{u_x}\right) = \frac{e\xi}{m}\left(\frac{x_1}{u_x}\right)\left(\frac{x_2}{u_x}\right) = \frac{e\xi}{m}\frac{x_1x_2}{u_x^2} \qquad \textbf{EM-5}$$

where x_2 is the horizontal distance traveled beyond the deflection plates. The total deflection $(y_1 + y_2)$ is proportional to e/m. Combining Equations EM-3, EM-4, and EM-5 and noting that $u = u_x$ for the undeflected beam, we have that

$$y_1 + y_2 = \frac{e}{m}\left(\frac{B^2}{\xi}\right)\left(\frac{x_1^2}{2} + x_1x_2\right) \qquad \textbf{EM-6}$$

Note the "direct" character of the measurement. Thomson needed only a voltmeter, an ammeter, and a measuring rod to determine e/m. It is also interesting to note that his original values of e/m from his first method, which averaged about 2×10^{11} C/kg, were closer to the present value of 1.76×10^{11} C/kg than those from his second method, 0.7×10^{11} C/kg. The inaccuracy of the results obtained from the second method was due to his having neglected the magnetic field outside the region of the deflecting plates. Despite this inaccuracy, however, the second method had the advantage of reproducibility and is considered the superior experiment.

Thomson repeated the experiment with different gases in the tube and different metals for cathodes and always obtained the same value of e/m within his experimental accuracy, thus showing that these particles were common to all metals. The agreement of these results with Zeeman's led to the unmistakable conclusion that these particles—called *corpuscles* by Thomson and later called *electrons* by Lorentz—having one unit of negative charge e and mass about 2000 times less mass than the lightest known atom, were constituents of *all* atoms.



EM-2 Deflection of the electron beam in Thomson's apparatus. The deflection plates are D and E in Figure EM-1. Deflection of the beam is shown here with the magnetic field turned off and the top plate positive. Thomson used voltages up to about 200 VDC between D and E. A magnetic field was applied perpendicular to the plane of the diagram directed into the page to bend the beam back down to its undeflected position.