Experiment IV:
Magnetic Fields and Inductance

I. References

Tipler and Mosca, *Physics for Scientists and Engineers*, 5th Ed., Chapter 27
Purcell, *Electricity and Magnetism*, Chapter 6

II. Equipment

Digital Oscillocope	Paper plates
Signal Generator	Foam plates
Differential Amplifier	Magnet wire
Resistors	Tape
Inductors	3 Neodymium magnets
LCR meter	Paper
Digital Multimeter	Business cards

III. Introduction to Magnetic Fields

Electrical currents generate magnetic fields, similar to the way that electrical charges generate electric fields. The magnitude of the magnetic field generated depends on the specific geometry of the wire in which the current is flowing, and sometimes in a complicated way, but for a given geometry, the magnetic field is directly proportional to the amount of current flowing through the wire.

Two equations describe the relationship between the electric current and the magnetic field that it generates. One is due to Biot and Savart:

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\ell}\times\vec{r}}{r^3} \qquad \text{IV-1}$$

where μ_0 (= $4\pi \times 10^{-7}$ Weber/meter-Ampère) is a constant, the permeability of a vacuum; *I* is the current in ampères (or Amps); $d\vec{\ell}$ is an elemental vector along the direction of current flow with the unit of length in meters; $\vec{r}$ is the position vector of the point at which the magnetic field is evaluated, with its origin at the position of the elemental length vector (its units are length in meters); and $\vec{B}$ is the magnetic field vector, which has units of Tesla or Weber/meter^2. You can think of this expression as a means of calculating the magnetic field generated by a current of magnitude *I* flowing along a small piece of wire with a length $d\vec{\ell}$ and a negligible cross-sectional area. The geometry is shown in Figure IV-1. This equation is often referred to as the Biot-Savart Law.

The second equation is

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I \ , \qquad \text{IV-2}$$

where $d\vec{\ell}$ is an elemental length vector along some closed contour in space, μ_0 is the permeability of the vacuum, *I* the net electrical current flowing through the area enclosed by the contour of which $d\vec{\ell}$ is an element and $\vec{B}$ is the magnetic field vector. This is called Ampère's Law. Whenever the symmetry of the situation makes it possible to choose a contour along which the magnitude *B* is a constant, Ampère's Law is a convenient way to compute the magnetic field generated by a current *I*.

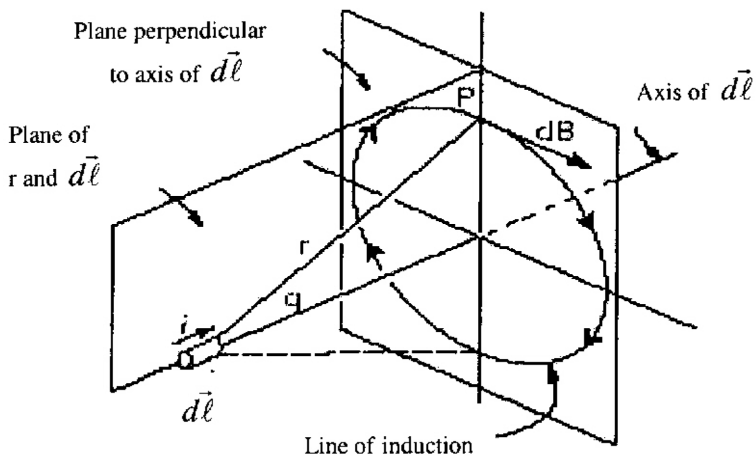

Figure IV-1: Magnetic field due to a current element

IV. Magnetic Field Strength along the Axis of a Circular Current Loop

By using the Biot-Savart Law and following an integration procedure given in some detail in most textbooks on Electricity and Magnetism, one can calculate the magnetic field strength, that is, the magnitude of the vector *B* generated by a current flowing in a circular loop of radius *a* at an arbitrary distance from the center of the loop *x* along the axis of the loop. See Fig. IV-2.
The result is

$$B = \frac{\mu_0 I}{2}\frac{a^2}{\left(a^2 + x^2\right)^{3/2}} . \qquad \text{IV-3}$$

If there are *N* loops close together, all carrying the same current, all having their centers nearly at the same place, and all having their radii nearly the same, a fairly good approximation for the magnetic field is given by

$$\vec{B} = \frac{\mu_0 NI}{2}\frac{a^2}{\left(a^2 + x^2\right)^{3/2}}\hat{x} . \qquad \text{IV-4}$$

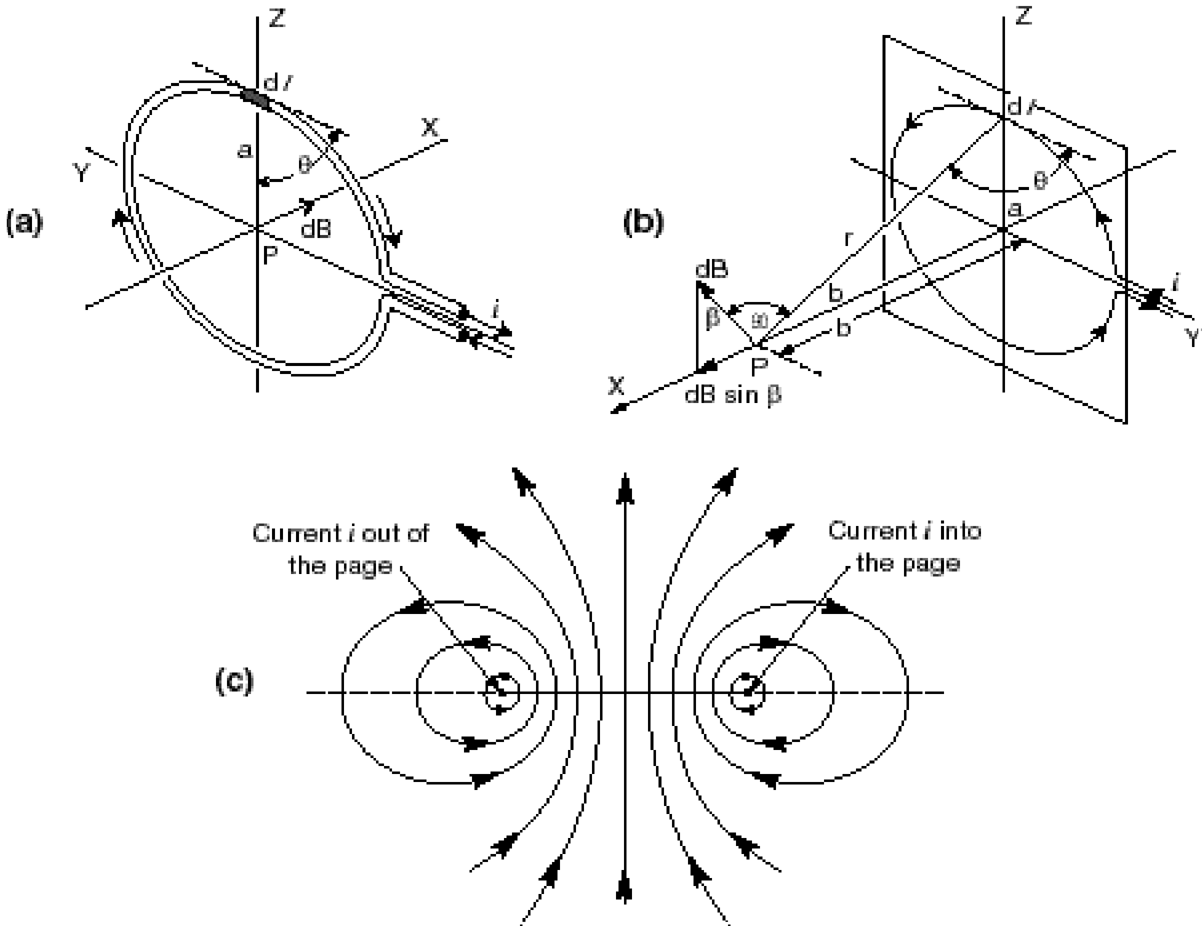

Figure IV-2: (a) Field at the center of a circular turn. (b) Field on the axis of a circular turn. (c) Lines of B for a circular loop.

V. Introduction to Inductors:

Devices that are considered to be inductors are generally something like a solenoid or a coil of wire. A current in a wire will generate a magnetic field, and the field depends upon the value and the direction of the current as well as the geometry formed by the wire (straight, loop, etc.). For a solenoid (see Figure IV-3), the field generated is along the axis of the solenoid, and reasonably constant within the solenoid as long as the solenoid is sufficiently long compared to its diameter that you can ignore edge effects.

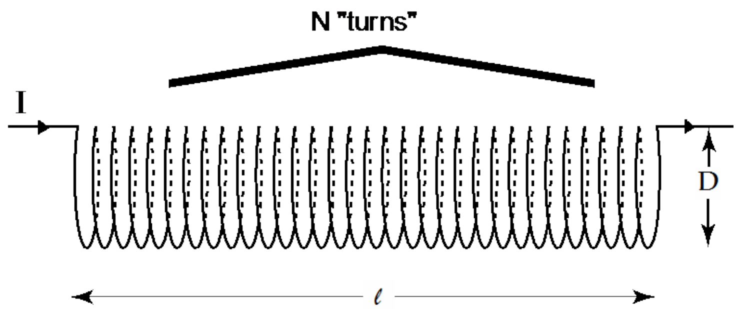

Figure IV-3: A coil of wire, or solenoid.

The field *B* generated by the current *I* in the solenoid is $B=\mu_0 NI/l$. The magnetic flux Φ_M is defined by the amount of the field *B* which is perpendicular to the area of the loop, $A=\pi D^2/4$. The total magnetic flux is given by the flux through each loop times the number of loops:

$$\Phi_M = N\vec{B}\cdot\vec{A}$$

where the vector part of *A* points perpendicular to the area itself. For the solenoid above, the field is perpendicular to the area, or $\vec{B}\cdot\vec{A} = BA$, so that the total flux is then simply given by

$$\Phi_M = NBA$$

Substituting $B=(\mu_0 N/l)\,I$ gives

$$\Phi_M = \frac{\mu_0 N^2 A}{l} I \qquad \text{IV-5}$$

So the magnetic flux through the device is a product of two things: the current *I*, and various quantities that are related only to geometry and constants. This means that the quantity

$$\Phi_M / I = \frac{\mu_0 N^2 A}{l} \equiv L \qquad \text{IV-6}$$

is independent of the current, and is a characteristic of the device's construction. The quantity (Φ_M/I) is called the inductance, *L*, and has units of flux (T-m^2) per current (Amps). This unit is called the "Henry". Note that Equation IV-6 was derived for a solenoid, but is in fact true of any device geometry.

Faraday's law states that a changing flux $(d\Phi_M/dt)$ will be "resisted" by the inductor such that a "back emf" is induced in the device, and since all devices have some inherent resistance, this will cause an induced current. This current will give rise to an induced magnetic field, and the direction of this field will be in a direction that will oppose the **change** in the flux (Lenz's Law). For example, if the flux *increases* in some direction, then the induced magnetic field will *increase* in the opposite direction to oppose the change. If the flux *decreases*, and is along some direction, the induced field will *increase* along the same direction, such that the net flux will (try to) remain constant.

If we differentiate Equation IV-5 above for the solenoid, we get

$$EMF = -\frac{d\Phi_M}{dt} = -\frac{\mu_0 N^2 A}{l} \cdot \frac{dI}{dt} = -L\frac{dI}{dt} = V_L \qquad \text{IV-7}$$

Since most inductors have very small resistances, most of the voltage drop across them is due to this "back emf", which comes from the combination of Faraday's and Lenz's laws. Note that all circuits have some amount of inductance, whether due to a "real" inductor, or due to the fact that the circuit contains wires, resistors, etc. which are subject to Faraday's and Lenz's laws. So understanding inductances is important for any circuit.

V. The RL Circuit With Step Input

Now let's look at a simple RL circuit, as shown in Figure IV-4. It consists of an input voltage V_{IN}, a resistor R, and an inductor L.

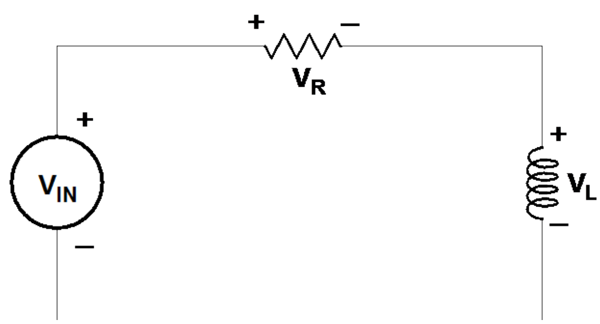

Figure IV-4: An RL circuit.

Conservation of energy, and Kirchhoff's Laws, require that $V_{IN} = V_L + V_R$. For the resistor, Ohm's Law allows us to make the substitution $V_R = IR$. Using the definition of inductance we can write $V_L = L dI/dt$ and thus come up with a first-order differential equation that describes the current in the circuit as a function of time.

$$V_{IN} = IR + L\frac{dI}{dt}. \qquad \text{IV-8}$$

If we imagine that V_{IN} is a constant V_0, and at $t=0$ it is "turned on" (goes from 0 to V_0), then once there is current flowing in the circuit, it is a DC circuit with $V_{IN} = V_0$ in Eq. IV-8. The procedure to solve this equation is the same as was used for the RC circuit. We can rearrange it to write

$$-(I - V_0/R) = L/R \cdot \frac{dI}{dt}.$$

Since the left hand side has units of amperes, the right side must as well. Therefore, the quantity L/R must have units of time. So we define the RL time constant to be $\tau=L/R$ and the initial current to be $I_0 = V_0/R$ so that we can rewrite the equation as

$$-(I - I_0) = \tau \frac{dI}{dt}.$$

Dividing both sides by $\tau(I - I_0)/dt$ and integrating gives us the current as a function of time.

$$I(t) = I_0 + ke^{-t/\tau} \qquad \text{IV-9}$$

where k is a constant of the integration, dependent upon initial conditions.

Figure IV-5 shows the initial condition $V(t=0)$ going from 0 to V_0. Just before $t=0$, when the voltage is zero, there is obviously no current. Just at $t=0$, when the voltage rises to V_0, the current begins to rise due to the finite resistance R in the circuit. Note that at these early times, the inductor simply acts like a piece of wire with a very small resistance, which we can neglect. The initial condition on the current is therefore $I(0)=0$, which means that the voltage drop across the resistor $V_R=0$ (since $V_R=IR$). Therefore, the entire voltage drop across the inductor V_L must be equal to the voltage V_0.

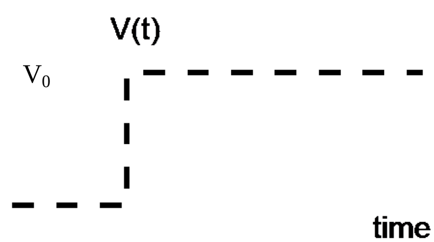

Figure IV-5: Input voltage as a function of time

The constant k can then be evaluated. Since the current through the circuit is initially 0 but changing rapidly, all the voltage in the circuit is initially dropped across the inductor.

$$V_L(0) = L\frac{dI}{dt}\bigg|_{t=0} = V_0.$$

Differentiating Eq. IV-8 and evaluating at $t=0$ gives $L \cdot dI/dt = V_0 = -k/\tau$, which gives a solution for k. With the definition $I_0 \equiv V_0/R$, we then have a complete solution for current as a function of time: $I(t) = I_0(1 - e^{-t/\tau})$. Using this expression, Ohm's Law and the definition of L allows us to write expressions for the voltage across the resistor $V_R(t)$ and the voltage across the inductor $V_L(t)$:

$$V_R(t) = V_0(1 - e^{-t/\tau}) \qquad \text{IV-10}$$

$$V_L(t) = V_0 e^{-t/\tau} \qquad \text{IV-11}$$

Note that the conservation of energy condition $V_{IN} = V_R + V_L$ is satisfied. Figure IV-6 shows the input voltage $V(t)=V_0$, $V_R(t)$, and $V_L(t)$. The *x*-axis is time in units of the exponential decay time $\tau \equiv L/R$. As indicated by $V(t)$, the input voltage goes from 0 to 1 Volt at time $t=0$.

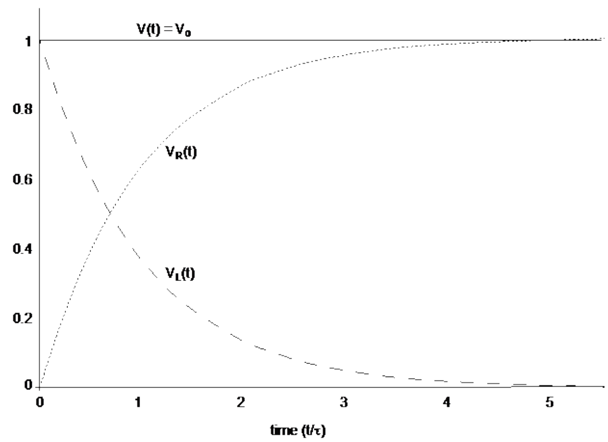

Figure IV-6: V_0, V_L and V_R as a function of time in an LR circuit for a square wave input.

VII. An Application: Magnetic Speakers

A familiar application of magnetic induction is the audio speaker. An audio speaker consists of an inductive coil affixed to the center of a paper cone or diaphragm which is suspended to allow it to move freely. Permanent magnets are then mounted in a configuration which is concentric with the coil such that the induced magnetic fields will either be aligned or anti-aligned. When a wave of a particular frequency is sent through the coil, the magnetic field changes polarity with the same frequency as the driving wave. The alternate attraction and repulsion between the coil and the permanent magnet drives the paper diaphragm with the frequency of the input wave. The vibrating diaphragm then produces sound waves by producing disturbances in the air of a corresponding frequency.

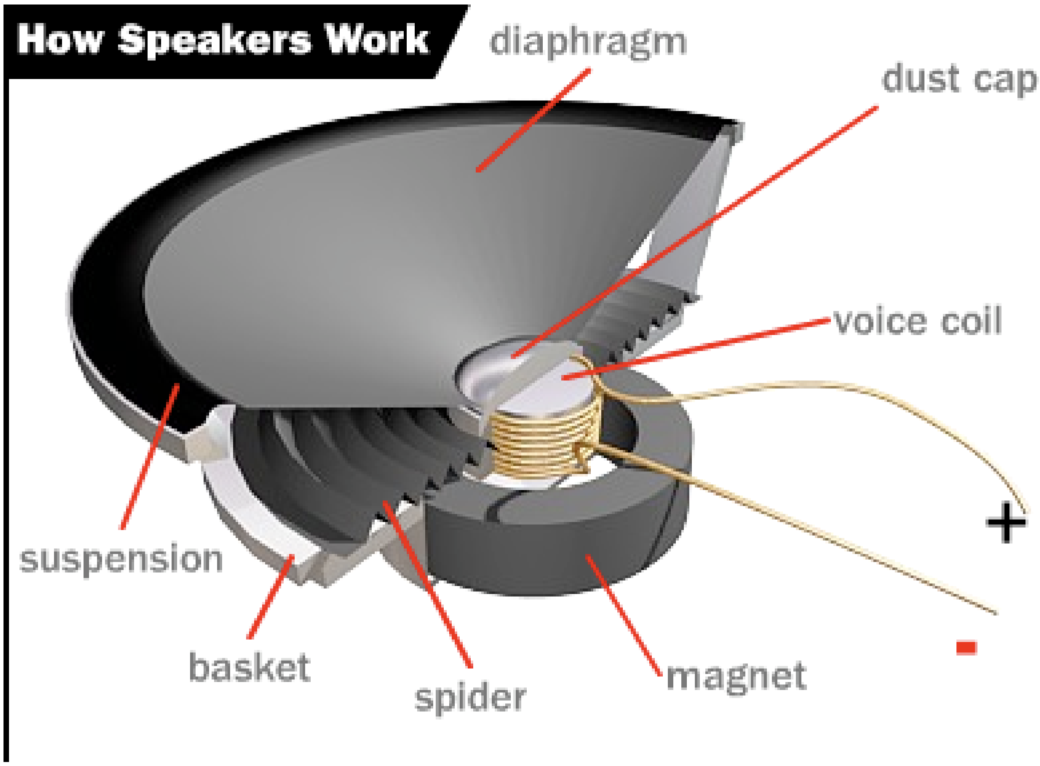

Figure IV-7: A typical audio speaker. Varying currents are induced in the coil, alternately attracting and repelling a permanent magnet as the polarity of the induced field changes polarity. The coil is affixed to a paper diaphragm, which it drives with a corresponding frequency.

VIII. Experiments

B. The RL Circuit

Construct the circuit shown in Figure IV-8. Use a 10 mH inductor and a 1 kΩ resistor: this gives a decay time $\tau = L/R = 10$ μs. Measure the components with the multimeter and LC meter, and calculate τ. The output from the frequency generator should be a square wave set to about 10 kHz, which corresponds to a period of about 100 μs. This means that the full exponential decay will be visible in the scope with the correct time base settings.

B.1: Trigger the scope externally using the TTL output of the waveform generator, and bring the output of the function generator and the voltage across the inductor into the two scope channels. Adjust the DC offset knob (pull out and turn to control the DC offset value) of the function generator to give the initial condition that $V = 0$ $(t < 0)$ and $V = V_0$ $(t > 0)$. Transfer traces of both waveforms, one above the other, and the difference $V_{IN} - V_L$ to your computer. Use the Math menu or EXCEL to get the difference signal. Include these plots in your report and compare the plots to your expectations. Be sure to save the data into a spreadsheet.

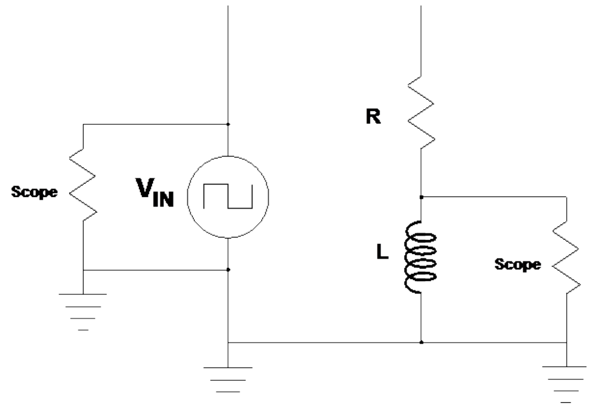

Figure IV-8: RL circuit to be used in Part B

B.2: With $V_L(t)$ displayed, estimate the RL time constant $\tau = L/R$ using the measurement tools on the oscilloscope as you did for the RC circuit in Lab 3. Again, transfer data to your computer and copy the data table to an EXCEL spreadsheet. Plot $ln(V_L)$ vs. time and obtain the L/R time. Compare to the value you got from the scope measurements and a direct calculation from the measured values of *R* and *L*.

B. Build a Speaker

You are to build a magnetic speaker. Detailed instructions are at the end of this section. Hook your speaker to the function generator and scan the audible frequencies to drive your speaker. You may increase the volume by turning up the amplitude on your function generator. Your device should be clearly audible! There is an example speaker at the back of the room if you need to look at it.

Call your instructor over and demonstrate the operation of your speaker. Make sure to record the procedure including the number of coils in your inductor. Make a diagram of your device. Take a photo of it with your phone. If you do not have a phone, call your instructor over and she will take a photo and send it to you. This should be included in your lab writeup, with all of the parts labeled, and the principle of operation clearly explained in your own words with any necessary equations.

Detailed speaker construction instructions:

(1) Roll a strip of paper (~ 2.5 cm width, full length) around the magnet, use two magnets stuck together (see Fig. S1). Use scotch tape to keep the paper from unrolling, only on the outermost layer. Roll a second strip of paper on the first roll, and tape the outermost layer. This will serve as a base for the coil.
(2) Start making your coil by wrapping the copper wire around the paper cylinder (see Fig. S2). Make **at least** 50 loops of wire. Leave about 25 cm of wire hanging out of the coil. Wrap some scotch tape around the coil, such that it holds together. Scratch off the outer layer (enamel) of the end of the wires using scissors or a razor blade. Remove the

innermost roll of paper and discard it (it has served is purpose of creating a gap between the magnets and the cylinder.)

(3) Glue the paper cylinder to the center of the **foam** plate. You may use either the hot glue gun (recommended) or scotch tape.

(4) Attach the two magnets to the base of your speaker (**paper** plate): place the magnets on **top** of your base, at the center, then place a third magnet **below** the base, directly below the two magnets. Use a paper plate as a base. Make sure to place the magnet at the center of the base.

(5) Fold the business cards in five parts, as shown in Fig. S3. Glue the folded business cards to the foam plate (see Fig. S4), you may use either the hot glue gun (recommended) or scotch tape.

(6) Lastly, glue the business cards to the paper plate, in such a way that the magnets are aligned to be **inside** the coil.

(7) Test the speaker: attach alligator wires to the end of each wire coming out of the coil, then attach the alligator wires to a battery, **one at a time**. You should see a very clear sign of inductance!

Fig. S1

Fig. S2

Fig. S3

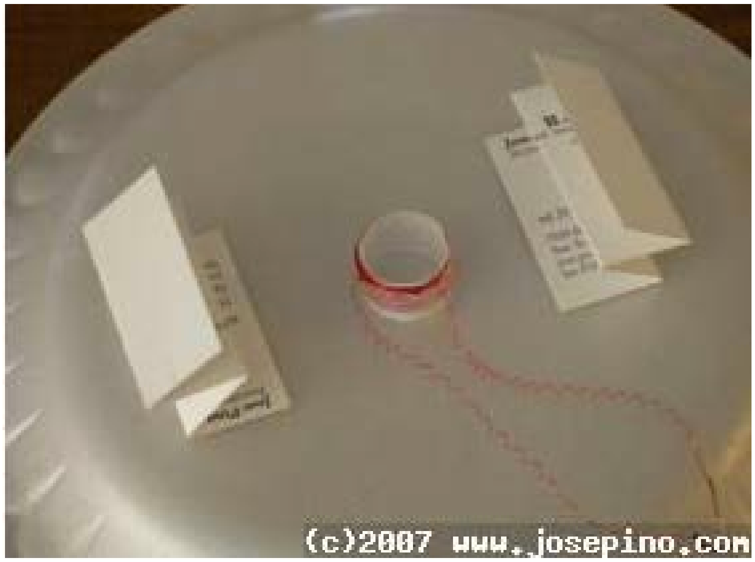

Fig. S4