
Experiment III: Capacitors and Low Pass Filters

I. References

Tipler and Mosca, *Physics for Scientists and Engineers*, 5th Ed., Chapters 25,28,29
Purcell, *Electricity and Magnetism*, Chapter 4

II. Equipment

Digital Oscilloscope
Signal Generator
Differential Amplifier
Resistors/Capacitors
LCR meter
Digital Multimeter

III. Introduction to Capacitors:

A capacitor is a conductor or set of conductors that can store electrical charge and thus store energy. They are ubiquitous in electronics, including radio applications. Usually a capacitor is constructed of two parallel surfaces (either two plates or two concentric cylinders) with a potential difference between them. The ability of a conductor to hold electric charge depends upon geometry of the conductor and also on the potential difference between the two plates.

For example, consider a pair of parallel plates each with an area A , separated by a distance d , and with a potential difference V between the two plates. This induces an equal and opposite charge density σ on both plates, and the electric field E is given by

$$E = \sigma / \epsilon_0$$

The electric field E is related to the voltage V by the equation $E = V/d$, so $V = \frac{E d}{\epsilon_0}$ or equivalently,

$$V = \frac{Qd}{\epsilon_0 A}$$

We define the *capacitance* C as the amount of charge present per unit voltage V applied:

$$C \equiv \frac{Q}{V} = \frac{\epsilon_0 A}{d} \quad \text{III-1}$$

The capacitance then depends only upon the geometry. This is true for any conductor of any geometry. This means that the ratio of charge to volts is constant for capacitors, which differentiates them from resistors, for which the constant ratio is between current and applied voltage applied. In this and subsequent labs, we will explore their use in both DC and AC circuits.

IV. The RC Circuit With Step Input

Introduction

In this class we will explore the DC behavior of components using a step input (a square wave from your function generator) as explained below. The signal at any given time is then a constant DC voltage, until it flips polarity and the process of charging or discharging starts again. This is distinct from the continually varying voltage in an AC (sinusoidal) circuit.

In this experiment we shall investigate the behavior of a simple circuit which contains a voltage source V_{IN} , a resistor R , and a capacitor C all in series as in Figure III-2.

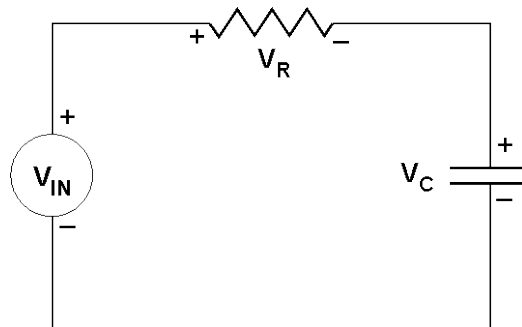


Figure III-1: An RC circuit with a voltage source V_{IN} .

Kirchhoff's Law tells us that the total change in potential around the circuit loop must be zero. Using this we can develop a first order differential equation to study the behavior of the voltage and current in the circuit.

$$V_{IN} = V_R + V_C = IR + \frac{Q}{C} = R \frac{dQ}{dt} + \frac{Q}{C}$$

It is straightforward to solve this equation by rearranging terms to get:

$$\frac{dQ}{Q - CV_{IN}} = -\frac{dt}{\tau}$$

where $\tau \equiv RC$, and Q is the charge on the capacitor (and whose derivative should be the current in the circuit). Note that the quantity τ has units of time. Integrating this equation gives

$$\ln(Q - CV_{IN}) = -t / \tau + k_1$$

where k_1 is a constant of integration. Taking the exponent of both sides gives us the behavior of the charge on the capacitor as a function of time. Applying the initial condition that $Q(0)=0$ allows us to replace the integration constant k_1 with $-CV_{IN}$, resulting in the following expression for $Q(t)$.

$$Q = CV_{IN}(1 - e^{-t/\tau})$$

Differentiating this expression once gives us the current in the circuit as a function of time.

$$I = \frac{dQ}{dt} = \frac{CV_{IN}}{\tau} e^{-t/\tau} = \frac{V_{IN}}{R} e^{-t/\tau} \quad \text{III -5}$$

We can now use Ohm's law for the resistor, $V_R = IR$, and the definition of capacitance $V_C = Q/C$ to find the voltages across the resistor and capacitor respectively:

$$\begin{aligned} V_R(t) &= V_{IN} e^{-t/\tau} \\ V_C(t) &= V_{IN}(1 - e^{-t/\tau}) \end{aligned} \quad \text{III-6}$$

Figure III-3 shows the voltage across the capacitor V_C and the resistor V_R as a function of time with the input voltage $V_{IN}=5$ Volts and $RC=1 \mu s$. We see in this figure that at first, all the voltage is dropped across the resistor, and then V_R falls exponentially to zero, while V_C climbs exponentially to V_{IN} . A capacitor is a "DC open" in the circuit, in the sense that no real DC current can get through. After a time that is long compared to the RC time constant τ , the current I in the circuit will go to zero, so V_R will go to zero as well.

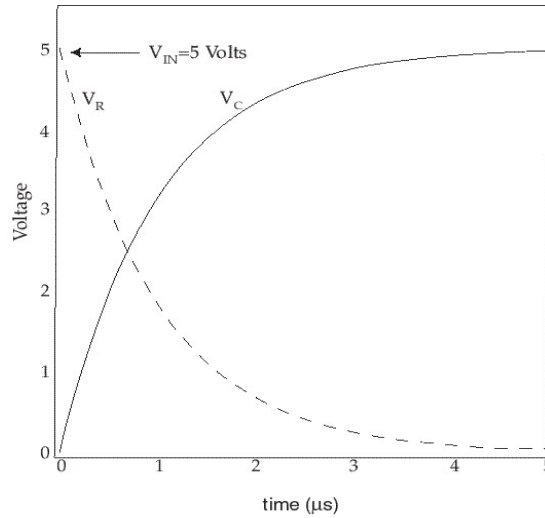


Figure III-2: V_C and V_R as a function of time.

Now, instead of using a DC voltage V_{IN} that is constant, you will use a function generator capable of producing a square wave, as in Figure III-4. This pulse train has a period T and frequency $f=1/T$, and amplitude V_0 . In essence, this is the same as if you were to take the DC voltage V_0 and every T seconds you invert the polarity to $-V_0$. The capacitor would then be charging, discharging, charging, discharging, etc. The resulting waveforms $V_{IN}(t)$ and $V_C(t)$ are shown in Fig. III-4

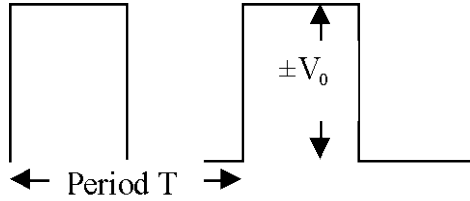


Figure III-3: The waveform of the input voltage you will be using in this experiment.

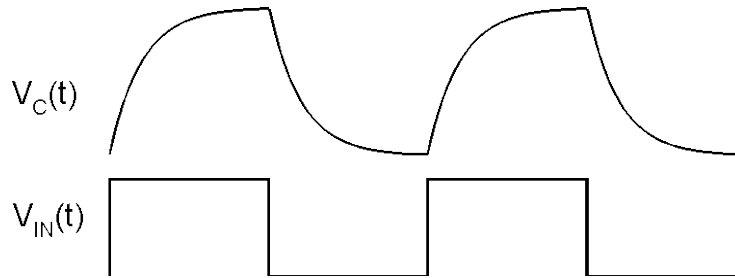


Figure III-4: The waveform of the voltage across the capacitor and the input voltage.

Applications of RC Circuits

If t is short compared to the time constant $\tau=RC$ and we consider the simple circuit in Fig. III-2, then $V_C \sim 0$ and the circuit equation becomes

$$V_R = IR = R \frac{dQ}{dt} = V_{IN}(t)$$

or, rewriting this as $dQ = \frac{1}{R} V_{IN}(t) dt$ and integrating gives $Q = \frac{1}{R} \int V_{IN}(t) dt$ which leads to

$$V_C = \frac{Q}{C} = \frac{1}{RC} \int V_{IN}(t) dt.$$

So for $t \ll RC$, then V_C is the integral of the input voltage V_{IN} and the circuit is an integrator.

If, on the other hand, t is long compared to the time constant $\tau = RC$, then the voltage across the resistor $V_R \sim 0$ and the circuit equation becomes

$$V_{IN} = V_C = \frac{Q}{C}$$

Differentiating the input voltage gives $\frac{dV_{IN}(t)}{dt} = \frac{1}{C} \frac{dQ}{dt} = \frac{I}{C}$, and since $I = \frac{V_R}{R}$, then the voltage across the resistor is the derivative of the input voltage:

$$V_R = RC \frac{dV_{IN}(t)}{dt}.$$

So for $t \gg RC$ the circuit is a differentiator of the input voltage with respect to time.

VI. Radio applications: filters

As we have seen above, capacitive circuits have a time constant associated with them that is defined by the product RC . If we attempt to change the polarity of a circuit much faster than this time constant, the capacitor will not fully charge or discharge before the polarity changes again, and the peaks and valleys in the wave will be smoothed out by the slow charging and discharging of the capacitor. Thus, high frequency signals are filtered out. Hence it can act as a “low-pass” filter, meaning that low frequency ($\ll 1/RC$) signals are passed through while high frequency ($\gg 1/RC$) signals are removed.

This can be used in several applications. One is the rectifier circuit we built in the last lab. If we wish to take a rectified signal and make a DC power supply out of it, we can simply add a capacitor to smooth out the peaks and valleys. Remember our rectified AM radio wave from the last lab? It still had the artifacts of the (relatively) high frequency AM carrier wave. How do we get rid of it and recover our original radio signal? Add a capacitor!

V. Experiments

A. The RC Circuit

Connect up the circuit shown in Figure II-5 (be careful with the grounds!!). First connect the function generator, resistor and capacitor together in a loop. Use a 10 k Ω resistor in series with a 0.1 μ F capacitor. Measure the resistance with a digital multimeter and the capacitance with an LCR meter *before connecting up the circuit*. You will need to use “BNC tee” and “BNC-to-banana” connectors from the function generator to the resistor. The outer shield of these connectors will be in direct contact with the third prong of the AC power cord of the function generator and thus are at “Ground” potential. Then connect CH1 of the oscilloscope to the function generator and CH2 across the capacitor. Again, you will need to use a “BNC-banana” connector and be sure that the “ground” side of the capacitor is connected to the outer shield of this connector. The ground of the oscilloscope and the ground of the function generator must be at the same potential (connected with no components in between). If you are unsure how to do this, ask.

Set up a 100 Hz square wave on of your function generator and look at the output in channel 1 (CH1) of the scope. Adjust the DC offset knob (pull out and turn to control the DC offset value)

of the function generator to give the initial condition that $V=0$ ($t<0$) and $V=V_0$ ($t>0$). Using a “tee” of the function generator output, and look at the voltage across the capacitor V_C using the other scope input channel (CH2). Be sure to pay attention to the grounds: if you hook the scope up “backwards”, you’ll be drawing current through the scope to ground instead of through your RC circuit. Notice that if you follow the diagrams correctly, all the grounds should coincide.

A-1: Display both V_{IN} and V_C simultaneously on the scope. Transfer traces of both waveforms, one above the other, and the difference $V_{IN}-V_C$ to your computer. Use the Math menu or EXCEL to get the difference signal. Include these plots in your report and compare the plots to your expectations. Be sure to save the data into a spreadsheet.

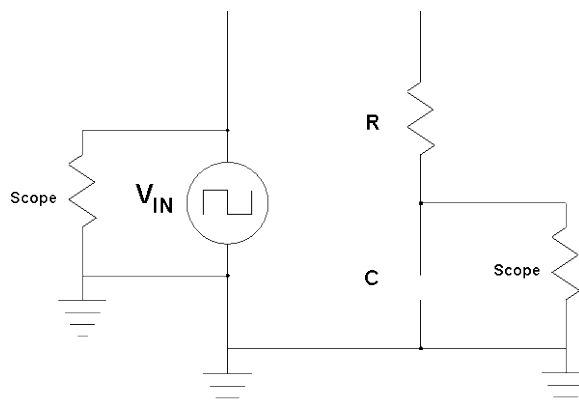


Figure III-5: Circuit to be used in Part A:

A-2: Change CH1 to now look at the voltage across the resistor R (V_R). You will need to use the “Instrumentation Amplifier” for V_R , otherwise you will connect the “downstream” side of the resistor to ground and effectively remove the capacitor from your circuit. The instrumentation amplifier has a buffer in it that isolates the (–) side of the input from the oscilloscope ground. Comment in your lab report on the signal you see in CH1: is it similar to the above signal you sketched (and saw on “CH1-CH2”)? Why should it be? On the scope, in “Math Menu”, look at the signal “CH1+CH2” and comment on what you see. Using the measuring tools on the oscilloscope, determine the RC time constant of the signal, τ . Recall (from Physics 275) that the decay time τ is the length of time it takes for the signal to drop to $1/e$, or ~ 0.37 of its value (or in half the decay time the signal height will have dropped by 0.61.). Note for this section you are looking at one decay that fills up a good portion of the scope screen.

A-3: Again, transfer data to your computer. Copy the data table to an EXCEL spreadsheet. Plot $\ln(V_R)$ vs. time and obtain the RC time constant from a fitted slope to the data. Compare your result to the measurements you made with the oscilloscope. Compare both measurements to a calculation of the RC time constant based on measured values of the capacitor and resistor used in the circuit.

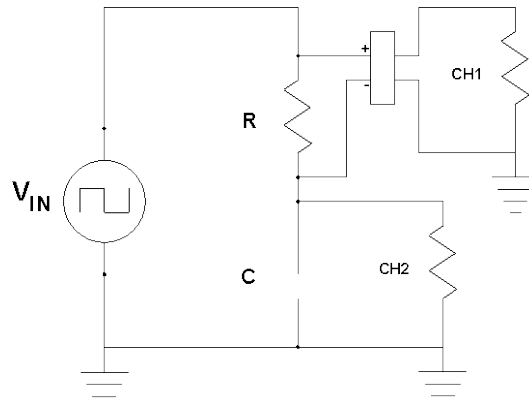


Figure III-6: Circuit with scope across the resistor using the instrumentation amplifier.

A-4: Vary the frequency of the generator by $1/10$ and by $\times 10$ so that you will be looking at the signals V_R and V_C for 10, 100, and 1000 Hz. What happens to the exponential decay constant? What happens to the amplitude of the pulse? Why?

B. Filtering

B-1: Connect up the full wave bridge you made in the last lab (shown below) using the germanium diodes, and measure the rectified signal on the load resistor with the oscilloscope. Because the resistor is not connected to ground on either side, you will need to use the differential preamp. It is suggested to start with a 1 kilo-ohm resistor and a 1000 Hz sine wave. Save an image of the oscilloscope output (be careful with grounds!).

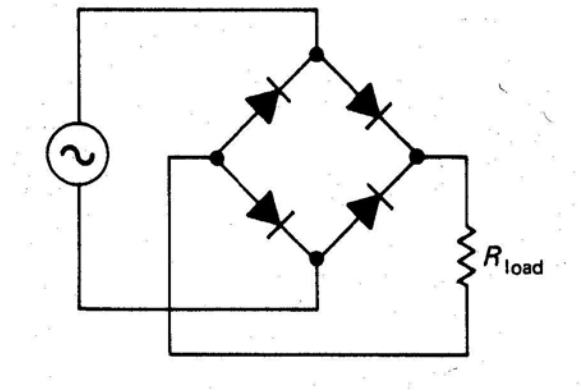


Figure III-7: The full-wave rectifier circuit from Lab 2.

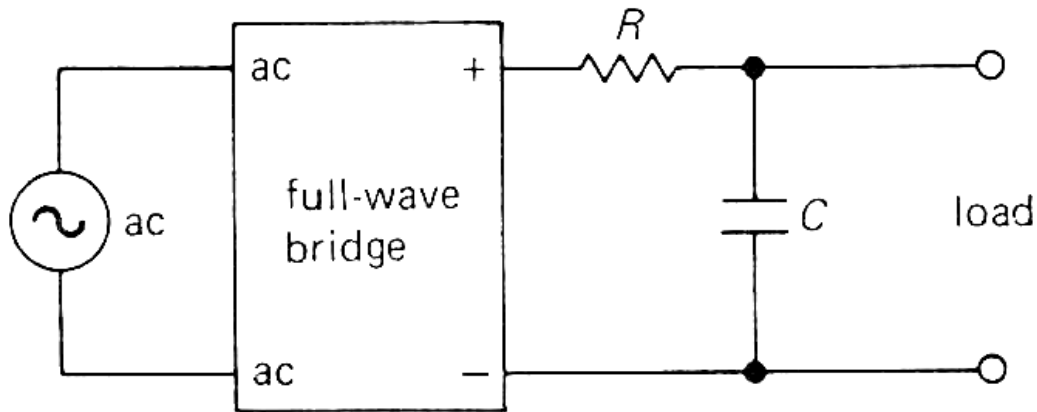


Figure III-8: Power supply circuit that converts the ac input signal into a dc voltage.

B-2: Now build the power supply circuit shown in Fig. III-8, with the R and C added as shown, and where your scope is the load. What value of capacitance do you need? How will your results depend on the value of capacitance chosen? We have variable capacitors available (boxes with the words “CS-301 precision capacitance substitute on them) so you can try a range of values. Make a screenshot. How do the results vary with frequency and capacitance (take measurements)? Comment in your spreadsheet.