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# Experiment I : Input and Output Impedance

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## I. References

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P. Tipler and G. Mosca, Physics for Scientists and Engineers, 5th Ed., Chapter 25  
D.C. Giancoli, Purcell, Physics for Scientists and Engineers, 4<sup>th</sup> Ed., Chapters 25, 26

## II. Equipment

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6-volt battery  
Digital multimeters  
Digital oscilloscope  
Signal generator  
Resistor board, resistor substitution box, misc. resistors  
Switch box  
Variable Voltage (up to 10 V) Power Supply

## III. Introduction

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When an electric charge is moved between two points in space, work may be done. The amount of work done is equal to the change in the potential energy of the charge. This difference in potential energy is given by the product of the difference in the electrical potential between the points and the magnitude of the electric charge. The difference in electrical potential  $V$  has an SI unit of volts.

The rate at which charge passes through some surface with a finite area is called the electrical current passing through that area. It is measured in Ampères. One Ampère of current is defined as one Coulomb of charge passing through a cross-sectional area per second. This is equivalent to about  $6 \times 10^{18}$  electrons passing per second. When a current of electrical charges is driven through some material system it is in most cases impeded by some sort of frictional drag. As a result, work must be done to move the charges against this retarding force, very much as air resistance must be worked against by the engine of a car. The current will only flow between two points in the material system if there is an electrical potential difference applied between those two points (providing the work). In many materials the current is directly proportional to the potential difference, i.e.

$$I = \frac{1}{R}V$$

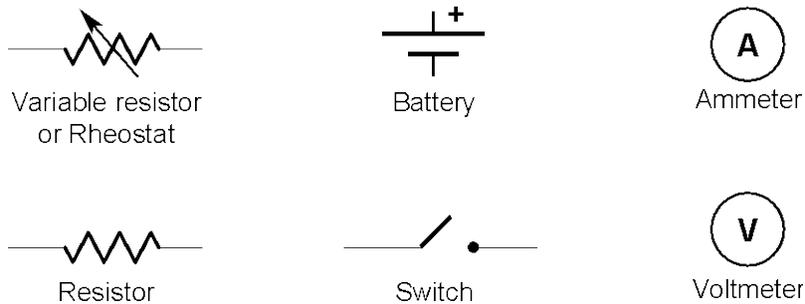
where  $I$  is the current in Ampères,  $V$  is the potential difference in volts and  $1/R$  is the proportionality factor. Usually this relationship is written  $V=IR$  and is called Ohm's Law. Here the proportionality factor  $R$  is called the resistance and is measured in Ohms. In many materials and devices the resistance does not change with the amount of voltage applied or the current passing through it, over a large range of both parameters, so it is a constant to a very good approximation. Materials with low resistance, in which electrons can move somewhat freely

with little impedance, are conductors, whereas materials in which the electrons are more tightly bound and cannot move freely are electrical insulators. Later this proportionality factor will be generalized in the concept of impedance.

## IV. Symbols

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Many of the components used in electrical experiments have standard symbols. Those required in this experiment are shown in Fig. (I-1).




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**Figure I-1**

## V. Electrical Circuits

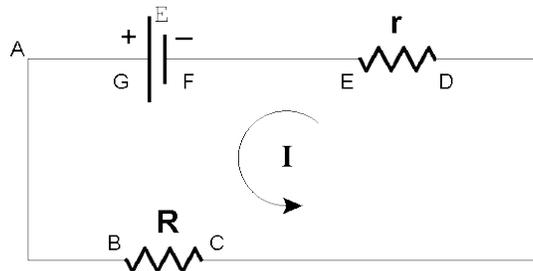
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It is very common to connect devices together with good conductors in such a way that a closed loop is formed. In passing round this loop the current may be split any number of times at the junction of several conductors or inside some device. However each new channel for current flow that is created at these junctions must rejoin another channel or even several channels at some other point, so that all loops close. All loops that are created must be closed so that current can flow. These connected systems are called electrical circuits.

### A. Kirchhoff's Rules

There are two very useful rules for understanding the behavior of electrical circuits.

**Rule 1:** In going round a closed loop in one direction the total change in potential must be zero.



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**Figure I-2**

Applying this rule to Fig. I-2 and assuming that the conductors joining the components have zero resistance, the potential differences between the lettered points in the circuit are given by:

$$\begin{aligned}V_A - V_B = \Delta V_{AB} = 0 & \quad \Delta V_{BC} = IR \\ \Delta V_{CD} = 0 & \quad \Delta V_{DE} = Ir \\ \Delta V_{EF} = 0 & \quad \Delta V_{FG} = -E\end{aligned}$$

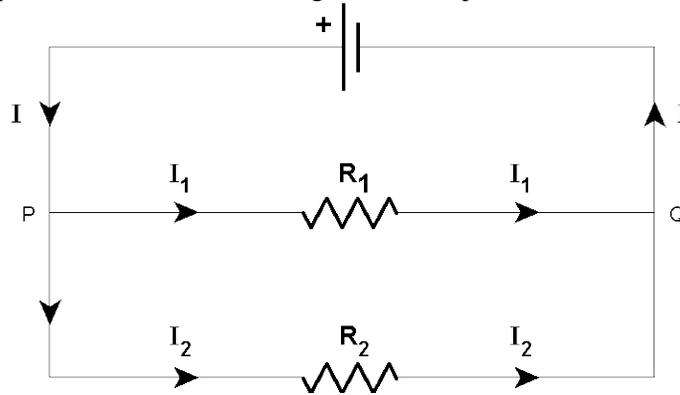
Summing all the differences we get

$$\Delta V_{AA} = IR + Ir - E = 0$$

which can be rewritten

$$E = I(R + r) .$$

**Rule 2:** Charge is conserved in any electrical circuit so that at any junction the current flowing into the junction is equal to the current flowing out of the junction.



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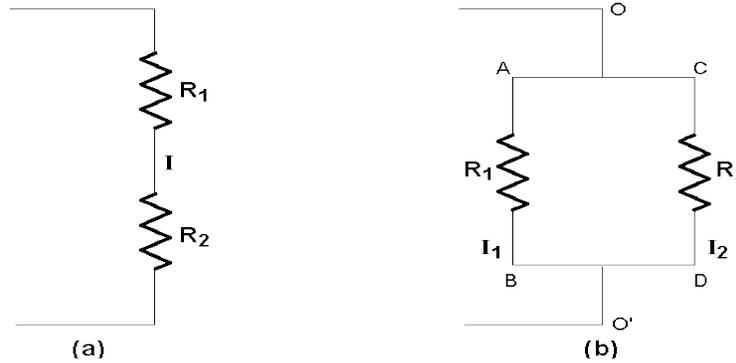
**Figure I-3**

When this rule is applied at the points P and Q of the circuit in Fig. I-3, we get

At P:	Current in = $I$	At Q:	Current in = $I_1$ and $I_2$
	Current out = $I_1 + I_2$		Current out = $I$

Both points yield the equation  $I = I_1 + I_2$ .

## VI. Computing the Effective Resistance of Networks of Resistors



**Figure I-4:** (a) Series connection.

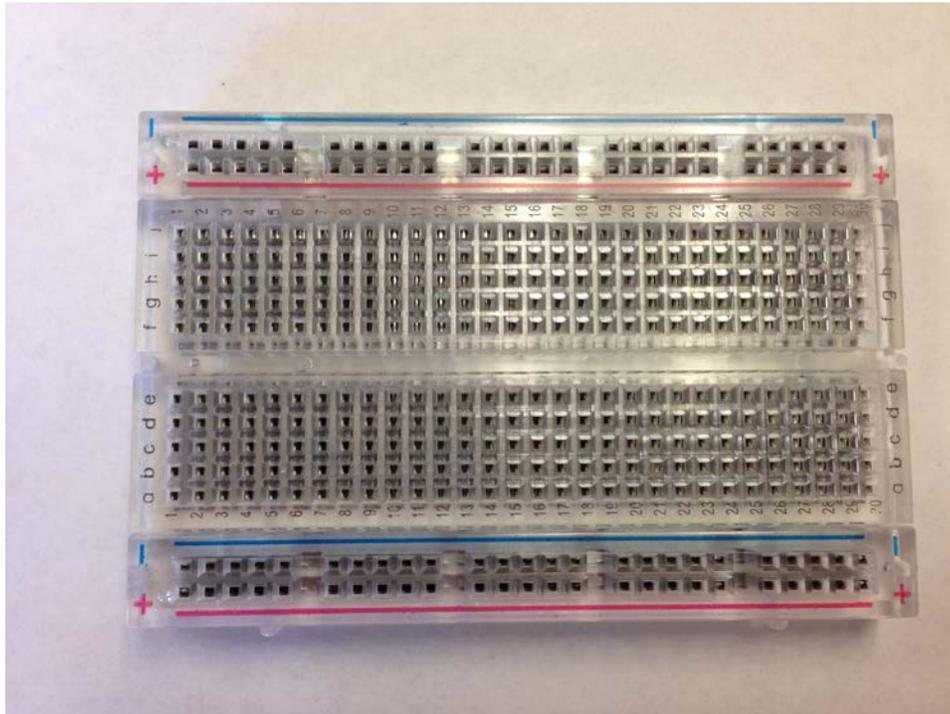
(b) Parallel connection

Series connection	Parallel connection
<p>Since current represents flow of charge and therefore must be conserved, the current in <math>R_1</math> must be the same as the current in <math>R_2</math>.</p> <p>Hence</p> $V_1 = IR_1$ $V_2 = IR_2$ <p>therefore</p> $V_1 + V_2 = I(R_1 + R_2)$ <p>which is equivalent to writing <math>V = IR</math></p> <p>where <math>V = V_1 + V_2</math></p> <p>and <math>R = R_1 + R_2</math></p> <p>i.e. two resistors connected in series are equivalent to one resistor whose value is equal to their sum.</p> <p>Generalizing, <math>R = \sum_i R_i</math></p>	<p>The potential difference <math>V</math> between <math>O</math> and <math>O'</math> must be the same whether we go along <math>OABO'</math> or <math>OCDO'</math>.</p> <p>Also conservation of current requires that</p> $I = I_1 + I_2$ $I_1 = \frac{V}{R_1} \quad I_2 = \frac{V}{R_2}$ <p>therefore</p> $I = V \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$ <p>which is equivalent to writing <math>V = IR</math></p> <p>where <math>R = \frac{R_1 R_2}{R_1 + R_2}</math></p> <p>i.e. two resistors are connected in parallel are equivalent to one whose reciprocal is equal to the sum of their reciprocals.</p> <p>Generalizing, <math>\frac{1}{R} = \sum_i \frac{1}{R_i}</math></p>

## VII. The Breadboard

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The breadboard is an important tool for prototyping electrical circuits. The purpose of the breadboard is to make secure connections between electronic components without soldering. This allows the components to be changed, and the circuit modified, easily.



Photograph of a breadboard

Some of the breadboards we will use for the course are clear plastic, which will allow you to see the electrical connections. The breadboard is pictured above. It consists of many holes in which to insert the leads on your components. On the top and bottom of your board, there are two rows labeled “+” and “-“. All of the holes in each of the “+” rows are connected to each other, and all of the holes in each of the “-“ rows are connected to each other. These rows are typically connected to the “+” and “-“ on your power supply. In the center, there are columns of five holes each, where each of the five holes are connected to each other, but not to the holes in neighboring columns.

## VIII. Experiment Part 1 – Resistors in Series and Parallel

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In this part, we will set up a few simple circuits with series and parallel resistors. Please find the resistors whose nominal values are 2.2 k ohm (2 of them) and 1.0 k ohm. Measure the value of your resistors with your multimeter (of course with an associated uncertainty!) and record them

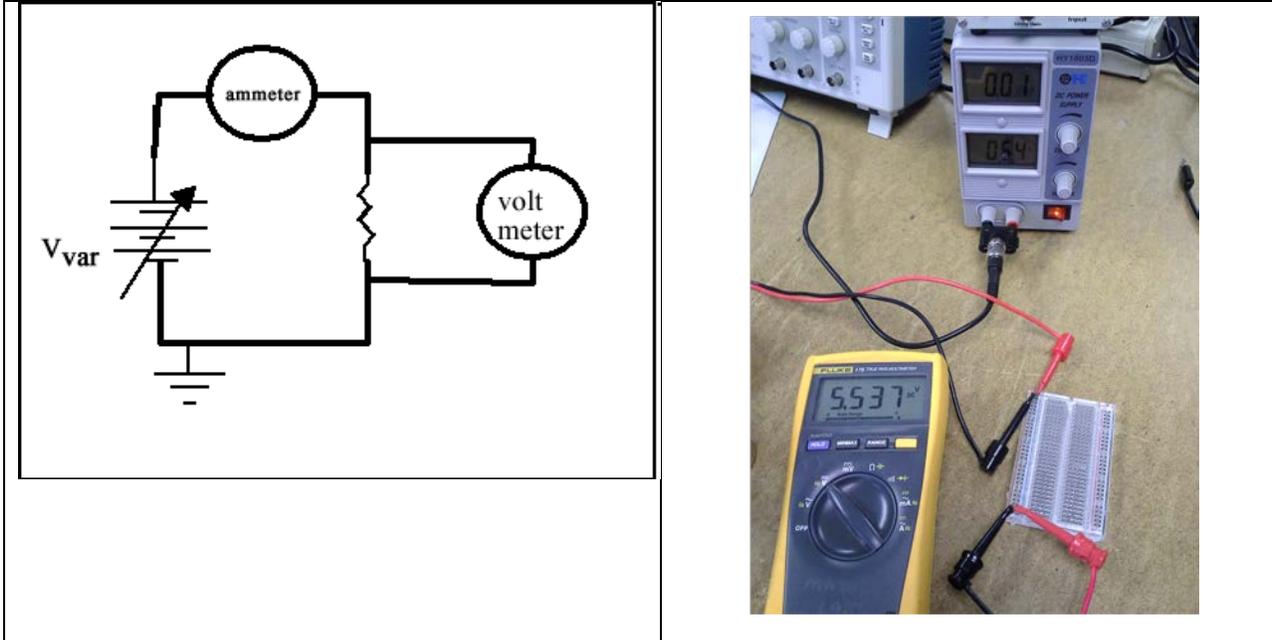
in your spreadsheet. Do they agree with the nominal values within uncertainties? Discuss with your instructor what might be the cause of discrepancies.

Try wiring the following circuits, using the bread board

- 1) Two resistors in series
- 2) Three resistors in series
- 3) Two resistors in parallel
- 4) Three resistors in parallel

For each combination, calculate the equivalent resistance of your circuit, and record the predicted values. Measure the equivalent resistance of each circuit using your multimeter (be sure and include an estimate of the error!).

After you complete these measurements, then, for the first circuit only, complete your circuit with your power supply (see picture below for a suggestion on how to do this). **Be VERY CAREFUL to start at the minimum voltage on the power supply and keep the current in the circuit below 400 mA** (otherwise the fuse will blow on the ammeter). Also, make sure that the current knob on the power supply is not turned all the way down. Finally, make sure that the ammeter is set to DC (not AC) current. Record the voltage versus current using your two meters for five different values of the voltage. Plot the results and put it in your spreadsheet. Determine the equivalent resistance of your circuit from the plot. Be sure to include statistical errors. Do not include the calibration (systematic) uncertainty for this part. Do your measured and predicted values agree within the uncertainties?



## IX. Experiment Part 2 - Internal Resistance and EMF of a Battery

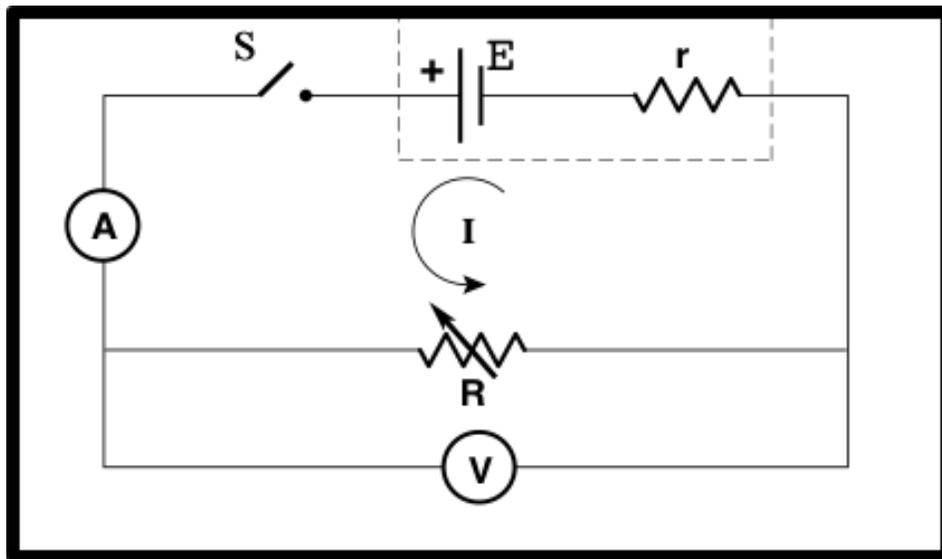
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Batteries produce an electrical potential difference through chemical reactions. If a connection is made between these two regions of the battery with some material, such as a wire or some other material that has finite resistance, a current will flow through the object and through the battery. The current is defined to be flowing from the side of the object that is connected to the region of the battery defined to have the more positive potential, through the object and to the side connected to the region of the battery with the more negative potential. The current flows from the positive to the negative potential. Inside the battery, the current flows from the more negative region to the more positive region.

Historically, a popular notion was that an electrical current needs to be pushed by some force, for example, through a piece of wire. The force was thought to be needed to overcome the resistance of the piece of wire to the flow of the current. This force was called the electromotive force (EMF), i.e. the force required to move the electric charges. To make the current flow required a battery, so the battery was thought to be the source of this electromotive force. But because the EMF is measured in volts, it is not a force but a potential difference. Part of this experiment is to measure the EMF of a battery, that is, the potential difference across its terminals.

When a current flows inside a battery it is also impeded by a small frictional force, so the battery has a small internal resistance that can also be measured with some care. Internal resistance is also referred to as the *output impedance* of the battery. The purpose of this experiment is to measure the internal resistance of a battery and its EMF using a known set of carbon resistors and Ohm's Law. The experiment is simple but the determination of the uncertainties on these two quantities is a bit subtle.

Connect up the circuit shown in Fig. I-5 (the resistances of the meters are not shown). Leave the switch open, have your instructor check your circuit before closing the switch.



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**Figure I-5**

Here A is a digital ammeter and V a digital voltmeter. The variable resistance R can be achieved by choosing various combinations of resistances on the resistor board or using your variable resistor box. It is important to note that the insertion of ammeter A introduces a small series resistance into the circuit and the insertion of voltmeter V introduces a large resistance in parallel with R. For the multimeter operating as an ammeter, measure its resistance  $r_A$  using a multimeter operating as an ohm meter. For the multimeter operating as a voltmeter, consult the manual for the meter to determine its resistance  $r_V$ . S is a switch that is normally open and should only be closed when you are taking a measurement. This helps avoid exhausting the battery. The internal resistance of the battery is denoted as  $r$ .

To perform the experiment, proceed as follows:

**A.** Measure V when R is a few 10s of ohms and when R is a few k ohms. What do you observe? Which will give more precise readings? **Discuss with your instructors before proceeding.** Now, measure V and I for various values of R (using your discussion with your instructor to help choose the values). Because the resistance of the circuit depends on temperature, and because this circuit produces a lot of heat, it is important to measure V and I at the same time. If you measure them at different times, you will be effectively measuring them for different circuits. There will be a random error associated with your ability to measure them at the same time. Try to estimate the size of this error. If the resistances of the meters, which are not shown, are neglected, then the equations describing the circuit are

$$V = IR, \quad E = I(R + r),$$

which can be combined and rewritten as

$$V = E - rI.$$

**B.** Put the data into Excel, plot V vs I (using the fitter). If there are sources of random error that are not negligible, include them as well. Fit the data to a straight line and determine the slope and offset  $r$  and  $E$ , along with their uncertainties. Is the chi-squared value from your fit reasonable? If not, discuss this with your instructor. Then, following instructions from your instructor, include the calibration (systematic) uncertainty in the fit result. Remember that the calibration uncertainty is not a random uncertainty, and has to be handled in a different way. The random uncertainties should be used in the linear fit, and the systematic uncertainties (here the calibration uncertainty) should be accounted for after you have extracted the fit parameters.

**C.** Now include the resistances of the meters in your analysis by including their effects in the circuit equations. By thinking about it (and some simple calculations), you should be able to deduce how the value of  $r$  and  $E$  would change if you included the effects of the meter resistance in your calculation.

## X. Experiment Part 3 – Input and Output Impedance

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All voltage or current sources (such as the battery in Part 1) have internal limitations which in many cases can be expressed as an output impedance and, in general, lower output impedance corresponds to the ability to deliver higher power to a load. In the earlier part of the experiment we measured the output impedance (internal resistance) of a battery as well as the internal voltage. Likewise, measuring devices have an input impedance which limits their ability to be non perturbing to the circuit being measured. In the earlier part we discussed the input resistance of an ammeter. The purpose of this part is to extend these concepts as well as provide a general method to think about and measure them. At this time we deal only with a case where the impedance can be specified as a resistance. The capacitive or inductive cases are not dealt with here. We will measure the output impedance of the signal generator.

The analysis here employs only simple applications of Ohm's law as described above. **For this part, only estimate random errors. Do not estimate or include systematic errors.**

### B. Output Impedance of the Signal Generator

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#### B.1. Experimental Setup (Fig I-7)

First hook up the equipment as shown in Fig I-7 using the resistor substitution box set on 1 kilohm. Be very careful about grounds (note the red lines in Fig I-8).

Set the signal generator on 1-2 kilohertz with a peak to peak amplitude of 2-5 volts (referred to as  $V_{out}$ ). You will not change these. You must set the oscilloscope on the proper scale and set the trigger menu on Chan 1 Internal and adjust the trigger options to get a good stable sine wave trace. Use the oscilloscope Measure menu to measure the peak to peak voltage.

The resistor box is called the *load resistance*  $R_{load}$  in the diagram and the signal generator is now *represented* as an ideal voltage source and an output resistance,  $R_{out}$ , which again is effectively inside the generator and which we will measure. Note that the signal generator  $V_{out}$  and  $R_{out}$  and  $R_{load}$  again form a series combination. For this part the input impedance of the oscilloscope is not important because it is much bigger than  $R_{load}$ .

#### B.2. Measurements

Now measure the peak to peak voltage on the oscilloscope Channel 1,  $V_{scope}$ , using the Measure function as you change the resistor box value going from 1 kilohm down to 30 ohm, taking about seven data points spaced out over that range. Be sure the voltage decreases as you decrease  $R_{load}$  and see the instructor if this is not so.

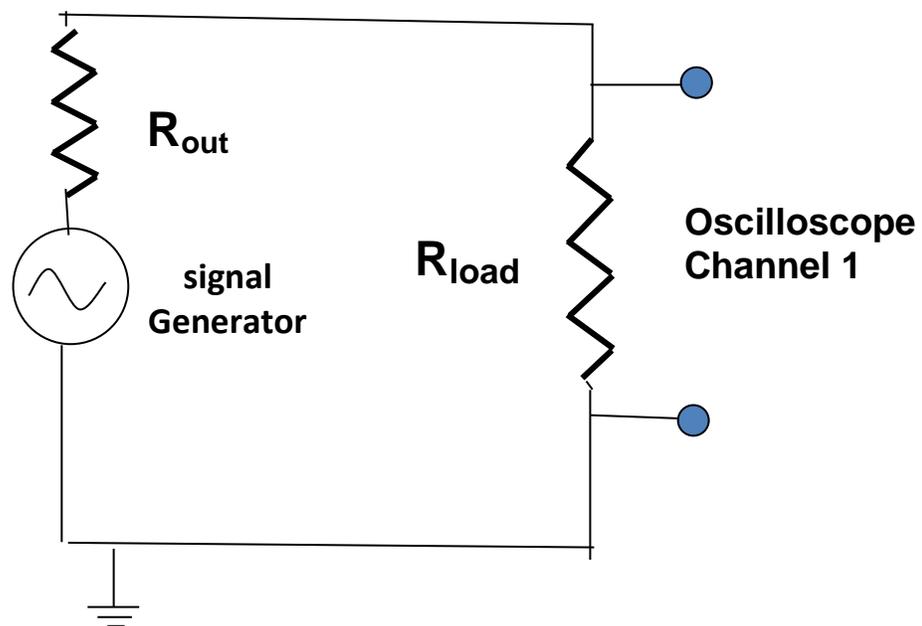
#### B.3. Analysis

1. If  $V_0$  is the unloaded voltage from the function generator, then using  $V_0 - IR_{out} - IR_{load} = 0$  and  $V_{CH1} = IR_{load}$  you can show:

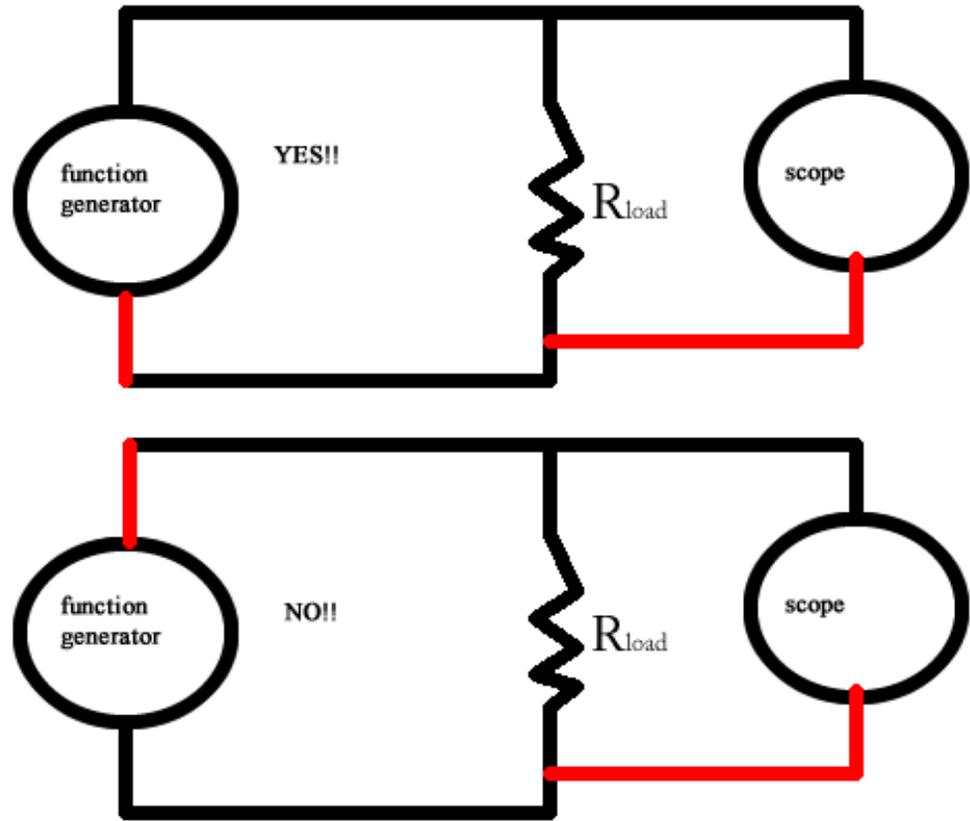
$$\frac{1}{V_{CH1}} = \frac{R_{out}}{V_0} \frac{1}{R_{load}} + \frac{1}{V_0}$$

2. Plot  $1/V_{CH1}$  versus  $1/R_{load}$  and their errors (remember to propagate the error in  $V_{CH1}$  to the error in  $1/V_{CH1}$ !)

3. Use `linear_fitter_276.xls` to fit your data and extract  $V_0$ ,  $R_{out}$ , and their errors.



**Figure I-7 : Output Impedance of Signal Generator**



**Figure 1-8: Ground connection Do's and Don'ts**