Homework #1

due Thursday February 1

(Note: I have explicitly typed the questions assigned from Hirose & Lonngren for the benefit of those who do not yet have a book. H & L #5 refers to a drawing in the book and I have tried to describe it in words. It is the same figure that I have drawn many times in lecture of the mass attached to a spring on a horizontal surface.)

1. Hirose and Lonngren Chapter 1 #4 (Show that functions

   (a) $x = A\sin\omega t$,

   (b) $x = A\sin\omega t + B\cos\omega t$

   (c) $x = A\cos(\omega t + \phi)$

   all satisfy

   $$M\frac{d^2x}{dt^2} = -kx \tag{1}$$

   provided $\omega = \sqrt{k/M}$. A, B, and ϕ are constants.)

2. Hirose and Lonngren Chapter 1 #5 (If a 1.5 kg mass attached to spring in the horizontal plane is displaced so that the spring is compressed 10 cm then released, twenty observations are observed in 1 minute. Find

   (a) The spring constant.

   (b) The equation describing the oscillation.

   (c) The energy associated with the oscillation.)

   (Answer: 6.6 N/m, (-10 cm)cos(2.1t), 3.3×10^{-2} *J)*

3. Hirose and Lonngren Chapter 3 #3. (Show that

   $$\cos x = \frac{e^{ix} + e^{-ix}}{2}, \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i}. \tag{2}$$

   Also display these relationships geometrically as vector diagrams in the $x - y$ plane.

4. To take successive derivatives of $e^{i\theta}$ with respect to θ, one merely multiplies by i:

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(Ae^{i\theta}\right) = iAe^{i\theta} \tag{3}$$

Show that this prescription works if the sinusoidal representation $e^{i\theta} = \cos\theta + i\sin\theta$ is used.

5. Write the following in exponential form i.e. $z = \rho e^{i\theta}$ and draw it:

   (a) $z = 1 + 2i$

   (b) $z = -3 + 2i$

   (c) $z = 5 - 3i$