Solutions to Homework 1

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Problem 1

a: Angular frequency $\omega = \sqrt{\frac{\kappa}{mass}}$, $\kappa = 800N/m$, $mass = 2kg$, therefore, $\omega = 20rad/s$.

b:Block released from rest A=20cm. displacement $y = 20\cos(\omega t)$, velocity $\nu = dy/dt$, Acceleration $a = d\nu/dt = -\omega^2 y$. For downward positive $y = 12cm$, $a = -4800cm/s^2$,upward negative $y = -12cm$, $a = 4800cm/s^2$; With Energy conservation, $E = 1/2\kappa A^2 = 1/2m\nu^2 + 1/2\kappa y^2$.we have $\upsilon = \pm\omega\sqrt{A^2 - y^2}$, for $y = \pm 12cm$, $\nu = \pm 320cm/s$

Problem 2

a Consider what happens when the mass is given a displacement $x > 0$,one spring will be stretched x and the other will be compressed x, they will each exert a force of magnitude $20\frac{N}{m} \times x$ on the mass in the direction opposite to the displacement. Hence the total restoring force is $F = -20\times x - 20\times x = -40N/m \times x$, $F = -\kappa x$ tell us the system has a spring constant $\kappa = 40N/m$. Hence,the period $T = 2\Pi\sqrt{\frac{mass}{\kappa}} = 0.54s$.

b:when the mass is displaced a distance y downward, each spring is stretched a distance y. The net restoring force is then $F = 2 \times (-20N/m)y$, Hence, again, from $F = -\kappa x$, we have $\kappa = 40N/m$, the same as in (a). $T = 0.54s$.

Problem 3

Mass m_2 shoots off when the spring stretched maximally and carrying kinetic energy K away from the system.so the amplitude of oscillation of m1 satisfies $1/2\kappa A^2 = 1/2m_1\nu^2$, where ν is the velocity at the equilibrium position.

To find ν, potential U of spring = maximum K_t of masses.
$\kappa d^2/2 = (m_1 + m_2)\nu^2$, giving $\nu^2 = (\kappa d^2)/(m_1 + m_2)$.
Then we have $1/2\kappa A^2 = 1/2m1\nu^2 = 1/2(\kappa d^2)/(m_1{+}m_2)$, then $A = d\sqrt{m_1/(m_1 + m_2)}$.

Problem 4

a: If the stick is rotated through a small angle θ, each spring is stretched a distance $L\theta/2$. Each spring causes a torque= $\theta/2 \times L/2$ with both torque in the same direction. The torque equation is
$-2\kappa\theta(L/2)(L/2) = I_{cm}\alpha$
where $I_{cm} = mL^2/12$, the momentum of inertia to certer of mass; $\alpha = d^2\theta/d^2t$
then $\alpha = -(6\kappa/m)\theta$. This is the equation for harmonic motion.
b) frequency $f = \sqrt{6\kappa/m}/2\Pi$.
c)The velocity reach the maxium when the stick passes the horizonal. Let θ_0 be the inital angle, so the maximum velocity=$(L/2)(2\Pi f)\theta_0 = L\theta_0(1.5\kappa/m)^{1/2}$.

Problem H1

Equation of motion is $\tau = I\alpha$, where τ is the external torque, I is the momentum of inertia, α is the angular acceration, and τ and I are about to the pivot point. TheContribution to τ is due to the rod and disk
$\tau = -mg(L/2)\sin\theta - Mg(R + L)\sin\theta$
where θ is a small angular displacement from the vertical.
$I = I_{rod} + I_{disk} = (mL^2)/3 + [(MR^2)/2 + M(R + L)^2]$.
where we used the paranal axis law to calculateI_{disk}.
Ror small θ, $\sin\theta \simeq \theta$, so the equation of the motion is
$-g[MR + ML + (mL)/2]\theta = [mL^2/3 + 3MR^2/2 + 2MRL + ML^2]\alpha$
so easilly to get the period $f = 1/T$, $(2\Pi f)^2$ is the coefficient of the θ term.
$T^2 = [4\Pi^2/g][(mL^2)/3 + (3MR^2)/2 + 2MRL + ML^2]/[ML + MR + (mL)/2]$.