

b) from a) $\Rightarrow T^2(x) \sin\theta \cos\theta = T_0 \cdot \omega x$

$$T^2(x) \frac{\tan\theta}{1 + \tan^2\theta} = T_0 \omega x$$

$$T^2(x) = T_0 \omega x \left(\tan\theta + \frac{1}{\tan\theta} \right)$$

$$\tan\theta = \frac{\omega}{T_0} x$$

$$\therefore T^2(x) = T_0 \omega x \left(\frac{\omega}{T_0} x + \frac{T_0}{\omega x} \right) = T_0^2 + \omega^2 x^2$$

$$T(x) = T_0 \sqrt{1 + \left(\frac{\omega x}{T_0}\right)^2}$$

3. $\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$ (1)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (2) \quad \Rightarrow \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E}) = -\frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (3)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (4)$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \frac{1}{c^2} \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E}) = -\frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{B}) - \Delta \vec{B} \quad \Rightarrow \quad \vec{\nabla} \times \vec{\nabla} \times \vec{B} = -\Delta \vec{B}$$

$\nabla \cdot \vec{B} = 0$

$$\therefore \Delta \vec{B} = \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}$$

4-dim. wave equation.

Similarly $\Delta \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$.

$$\Delta = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$$