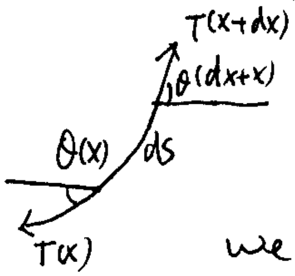
$\lambda_{2,3}=3, \quad \Rightarrow \omega=\sqrt{3}\,\omega_0$

correspond to $A_1=0, \; A_2=-A_3$

or $A_2=0, \; A_1=-A_3$

$A_3=0, \; A_1=-A_2.$

2. a). we use an elementary method to solve this problem. The conditions for static equilibrium are



$$\begin{cases} T(x+dx)\cos\big(\theta(x+dx)\big)-T(x)\cos\theta(x)=F_x=0 & *1 \\ T(x+dx)\sin\big(\theta(x+dx)\big)-T(x)\sin\theta(x)=w\,dx. & *2 \end{cases}$$

we rewrite $*1, *2$ as

$$\frac{d}{dx}\left[T(x)\cos\theta(x)\right]=0$$

$$\frac{d}{dx}\left[T(x)\sin\theta(x)\right]=w\cdot dx$$

integrating both equs, we obtain

$$T(x)\sin\theta(x)=C_0+w\cdot x$$

$$T(x)\cos\theta(x)=C_1$$

Boundary condition: $x=0 \;\; \theta=0 \;\;$ so $C_0=0$. $\cancel{T_0}$.

$x=0 \;\; T=T_0 \;\; \Rightarrow C_1=T_0.$

$$\therefore \left.\begin{aligned} T(x)\sin\theta(x)&=wx \\ T(x)\cos\theta(x)&=T_0 \end{aligned}\right\} \Rightarrow \tan\theta=\frac{wx}{T_0}$$

$$\tan\theta(x)=\frac{dy}{dx}=\frac{w}{T_0}\cdot x. \;\Rightarrow\; y=y_0+\frac{w\cdot x^2}{2T_0}$$