Using the definitions of curl and divergence, and $\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$.

From $\vec{\nabla} \times \vec{E} = 0 \Rightarrow \quad -\frac{\partial E_z}{\partial x}\vec{a}_y + \frac{\partial E_y}{\partial x}\vec{a}_z = 0. \quad *1$

$\vec{\nabla} \cdot \vec{E} = 0 \Rightarrow \quad \frac{\partial E}{\partial x} = 0.$

$*2$

$\therefore *1 \Rightarrow E_z$ and E_y must be constants independent of $x \cdot y \cdot z$. however boundary conditions requires they should both vanish on the conducting plates; otherwise the continuity of tangential $\vec{E}$ would require the existence of $\vec{E}$ field within the conducting plates. This $\vec{E}$ field in turn would exert a lorentz force on the charge, causing them to move, contrary to the assumed stationary character of the charge distribution. The condition may be satisfied only if

$$E_y = E_z = 0.$$

and $*2 \Rightarrow \quad E_x = \text{constant } C$

$$D_x = \varepsilon_0 E_x = \varepsilon_0 C.$$

boundary condition : $\vec{n} \cdot (\vec{D}_2 - \vec{D}_1) = \rho_s \qquad E_1 = 0 \Rightarrow D_1 = 0$

$$\Rightarrow D_x = -\rho_s$$

Similarly for lower plate, $C = -\rho_s / \varepsilon_0$.

Simillar procedure to obtain $\vec{H}$. since no currents, $\vec{H} = 0$

and at any point between the plates, $\vec{E} = C\vec{a}_x = -\frac{\rho_s}{\varepsilon_0}\vec{a}_x$.