Solution to #W3

1. Choose a circular path of radius a in the $z=0$ plane, along which E_ϕ must be constant by symmetry, then

$$emf = \oint \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$= 2\pi a E_\phi = -bB_0 e^{bt} \pi a^2$$

This is true for any path in the $z=0$ plane enclosing the same area. replace a by r.

$$\vec{E} = -\tfrac{1}{2} b B_0 e^{bt} r \bar{a}_\phi$$

Now attempt to obtain the same answer from

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$(\nabla \times \vec{E})_z = -bB_0 e^{bt} = \frac{1}{r} \frac{\partial (rE_\phi)}{\partial r}.$$

$$\vec{E} = -\tfrac{1}{2} b B_0 e^{bt} r \bar{a}_\phi$$

2. The total displacement current I_D is

$$I_D = \varepsilon_0 \frac{\partial E}{\partial t} \pi R^2$$

$$= 8.9 \cdot 10^{-12} \cdot 10^{13} \cdot 3.14 \cdot (0.1)^2 = 2.8\ A.$$

From Ampere's Law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{s}$$