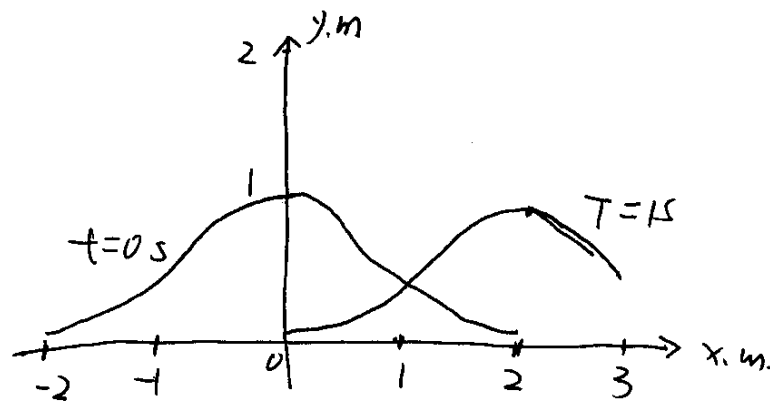4. a)

b) we begin with the proposed solution $f(x,t)=Ae^{-B(x-vt)^2}$

$$\frac{\partial f}{\partial x} = -2AB(x-vt)e^{-B(x-vt)^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \left[-2AB + 4AB^2(x-vt)^2\right]e^{-B(x-vt)^2}$$

$$\frac{\partial f}{\partial t} = 2ABv(x-vt)e^{-B(x-vt)^2}$$

$$\frac{\partial^2 f}{\partial t^2} = \left[-2ABv^2 + 4AB^2v^2(x-vt)^2\right]e^{-B(x-vt)^2}$$

$$= v^2\frac{\partial^2 f}{\partial x^2}$$

$$\therefore \frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$